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PRACTICAL NOTES
FOR
ELECTRICAL STUDENTS.

VOL. I.

BY

A. E. KENNELLY & H. D. WILKINSON.

PRACTICAL NOTES

FOR

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VOLUME I.

LAWS, UNITS, AND SIMPLE MEASURING INSTRUMENTS.

BY

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AND

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PREFACE.

THIS book is mainly a reprint of a series of articles which appeared in *The Electrician* under the title of "Letters for Learners and Unprofessional Readers." The series was begun in November, 1887, by Mr. A. E. Kennelly but subsequently relinquished on his appointment to Mr. Edison's laboratory in America. In the beginning of the following year, at the Editor's request, I undertook the task of carrying them on under the same title, although I found the last two words in the title somewhat ambiguous. If a general and popular style of writing was looked for by the unprofessional readers, I cannot hope to have pleased them; but if, while engaged in other professions, they wished to become acquainted with some of the laws underlying this rapidly-growing science, then I may hope that they have found something worthy of their attention and interest.

To serve this purpose, I started with the good resolution to exclude all mathematical expressions, and endeavour to put into words anything expressing a relation between quantities. Unfortunately, this good resolution was speedily broken, although not for the want of trying to keep it.

The reader may, however, be reassured by the fact that only simple equations are used, and the three simple functions of angles. The only class of men that I could think of in another profession who could be sufficiently interested to follow up such a series were those learned gentlemen who square up some of our little differences for us not very far from Temple Bar.

On the other hand, to those who are, in common with myself, students of Electrical Science and Practice, I may hope that, in whatever branch of its application they are engaged, some good may accrue through these pages ; not on account of anything which might be termed novel, for this will not be found, but on account of the pains I have taken to explain in plain language some of the fundamental laws and processes, and the more modern terms in use at the present day.

The great disparity between the lengths of some of the chapters may strike those who are accustomed to write books, and divide up equally the matter treated upon, as ludicrous. This, however, must stand as it is, the subjects handled having been developed as it was thought necessary at the time of writing. The length of the last chapter somewhat appalled me, but I could not well leave the subject without adding what I considered to be of use and interest under that heading, although at the expense of being somewhat theoretical at times.

The experiments mentioned were conducted at the School of Electrical Engineering in Hanover Square, London, and

I have to thank Mr. Lant Carpenter, Mr. L. Drugman and the Students there who assisted me, amongst whom I recollect with great pleasure Mr. Webb-Watts, Mr. Ernest Tidd, Mr. Thomson and Mr. Blakeley.

It will be observed that the present volume contains several hints of continuation, and, indeed, by itself, cannot be called a complete treatise. I hope, therefore, as soon as time and opportunity permit, to take the matter up once more, and continue the work under its new title.

H. D. WILKINSON.

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PRACTICAL NOTES

FOR

ELECTRICAL STUDENTS.

I.—LAWS, UNITS, AND SIMPLE MEASURING APPARATUS.

CHAPTER I.

1. Early Ideas.—Notwithstanding the numerous applications of electricity, our knowledge of its nature remains of the most scanty description. In the earlier stages of investigation it was supposed to have a material existence in the form of an extremely attenuated fluid that could readily permeate conducting substances, but which found obstruction in passing through non-conductors; in the same way that liquid might pass freely through fibrous material, but be completely arrested by a densely solid barrier. Hence the terms "Electric Fluid," "Current," &c., which are still retained, not only on account of their widespread acceptation at a time when their application was more accredited, but also for the convenience with which they serve to connect our ideas in dealing with the more intricate conceptions of modern theory. There is, as we shall see, considerable analogy between the laws of the flow of water and the laws of the transmission of electricity; but in employing the terms suggesting fluids it is necessary to remember that they are only convenient symbols by which we represent and express a totally different class of phenomena.

2. Electrical Energy.—It is now generally agreed that electricity is immaterial in its nature, but is an inherent property of matter, through the medium of which, as with heat, gravitation, or chemical affinity, matter may acquire energy and exhibit force. This conclusion is supported by many considerations. For example, if electricity were matter we should expect it to exert gravitation like all other known forms of matter, but no one has ever been able to detect any increase in the weight of a condenser after charging it. On the other hand, every evidence of the presence of electricity, since its very first recorded discovery by Thales, in the attractive power of rubbed amber, has been, and is, through some manifestation of force; while it is always found that the production of a certain quantity of electric force requires the expenditure of a corresponding quantity of energy in some other form. Thus, to obtain electricity from a frictional machine or a dynamo, we have to turn the handle of the one or the armature of the other, and if we leave out of consideration the energy wasted in friction and heat, the power of the electricity produced will be the exact counterpart of the power mechanically expended in the turning process.

3. Electricity Produced by Chemical Energy.—When we obtain electricity from a battery, however, we expend no power in direct mechanical action, but we find zinc and other chemicals consumed. In this case it is chemical force and energy of combination which is expended. The zinc combines with the oxygen of the water in the cells, and is thus practically burnt; while for every grain weight of zinc usefully consumed a definite quantity of electric power is generated.

4. Electromotive Force.—In different kinds of batteries the same weight of zinc consumed may yield very different quantities of electrical energy, since the chemical affinity of the elements varies in different combinations of metals and solutions. The greater the resulting chemical affinity, the greater will be the energy expended in the union of a given quantity of the elements, and the greater is the corresponding quantity of electrical energy developed—in other words, the

greater the *electromotive force* of the cell. This electromotive force is conveniently expressed by the abbreviation E.M.F., and is practically measured in definite units, called *volts*.

5. **Requirements in a Good Cell.**—A good type of galvanic cell should fulfil the following requirements :—

(1.) The total useful chemical affinity, and thus the E.M.F., should be as great as possible, and constant under all rates of current supply.

(2.) None of this energy of affinity should waste itself in forming combinations while the cell remains idle. In other words, there should be no loss of material by local action.

(3.) The cell should offer in its own elements and solutions as little obstruction as possible to the liberation of the electricity it develops—that is to say, its internal resistance should be as low as possible.

(4.) The materials for its consumption should be cheap and readily obtainable.

(5.) The cell should require no inspection or supervision to keep it in good order until all the energy of its chemical affinities is exhausted.

(6.) The form and dimensions of the cell should be convenient, and no noxious chemical products should be formed in it by action.

No battery fulfils all these conditions with any degree of completeness, but we may discuss one or two of the more general forms and compare them by this standard.

6. **Simple Cell.**—The simplest form of cell consists of a plate of zinc and a plate of copper plunged in water. As water is not a very good conductor, it may with advantage be acidulated, or rendered saline by the addition of common salt. No electrical action takes place if the metals are pure as long as the plates are kept apart, but the moment they are brought into contact, either in the water or above its surface, a current will immediately be generated. It is not necessary

that the plates themselves should be made to touch each other, for the wire F G H (Fig. 1), usually copper connecting wire, may be considered as an extension of the copper plate, making contact with the zinc at the point H.

7. **Difference of Potential.**—The contact of these two metals is said to set up a *difference of potential*, and to be the source of the E.M.F., although the explanation of this observed fact cannot be said to be known. Difference of potential is with electricity precisely like what difference of level is with water; and just as water flows in virtue of difference of level from the higher to the lower, and the greater the difference of level the greater the force or head of water, so electricity flows in virtue of difference of potential from the point of higher to the point of lower, and the greater this difference of potential the greater the electromotive force. In fact, difference of potential constitutes electromotive force.

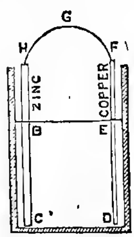


Fig. 1.

8. **Direction of Current.**—The difference of potential set up by the contact of the copper wire with the zinc causes the latter to become positive and the copper wire negative; in other words, the zinc takes the higher potential and the copper wire the lower, and an electromotive force is thus determined, generating a current from the zinc plate to the copper wire. There are obviously two paths by which this current from B can reach H—one direct across the junction, the other circuitous through the water, copper plate, and wire, B E F G. The current is prevented by the E.M.F. itself from taking the direct path across the surface of contact, and is therefore compelled to take the second and longer route. It is therefore

important to notice that the zinc plate, being positive to the copper wire as regards the junction H, is usually spoken of as the *positive element*, and the copper as the *negative element* of the cell; while externally, because the current is flowing in the wire F G H from F to G, the potential at F, though lower than that at B or E, is higher than that at H, and is therefore positive relatively to H. Consequently, on cutting the wire at G, the portions H G on the zinc and F G on the copper are called the *poles* of the cell, and F G, being positive to H G, is called the positive pole. So that, while the zinc is internally the positive element, the copper is externally the positive pole.

9. **Chemical Action.**—If the difference of potential due to contact at H were only temporary, the current flowing from B round to H would re-establish equilibrium and cease, just as the flow of water from a higher to a lower level tends to equalise the level; but it is generally considered that the current having once started is maintained by the chemical energy of the materials in the cell. Thus, in our typical case represented in Fig. 1, after the current has commenced to flow, bubbles of gas will be seen to form on the copper plate, and to rise to the surface of the water. The water is being decomposed into its constituents, oxygen and hydrogen, the oxygen attacking the zinc and uniting chemically with it, forming a coating of zinc oxide on its surface; while the hydrogen, not being able to form any chemical combination, forms in bubbles on the surface of the copper plate, these bubbles rising to the surface of the water as their buoyancy overcomes their attraction to the metal. If the water has been previously acidulated by the addition of a little sulphuric acid, the insoluble coating of oxide on the zinc plate is converted by the acid into sulphate of zinc, which, being soluble, disappears into the water, leaving the fresh surface of the zinc free. Thus in the action of the cell we find that the zinc plate is gradually consumed and loses weight, sulphate of zinc accumulates in the solution, the copper plate undergoes no change, and bubbles of hydrogen rise on its surface.

10. Polarisation.—Such a cell fairly satisfies the above requirements except the first two; in fact, this was the original form of the galvanic cell, and the ancestor of all the numerous types in modern use. Its great weak point lies in its low and variable E.M.F., for the hydrogen bubbles in contact with the copper plate set up a counter E.M.F., opposing the current maintained by the original E.M.F. due to the contact of the zinc and copper. This phenomenon of gaseous contact and counter E.M.F. is commonly called *polarisation,* and the amount of polarisation varies with the quantity of hydrogen in contact; consequently, the resultant E.M.F. is very variable. The first attempt to overcome this difficulty was roughening the surface of the copper plate, so that the bubbles of gas were more quickly liberated upwards, and thus the limits of the variation in the E.M.F. were kept more confined. The early Smee cells were of this nature, but it was not until the invention of the Daniell cell that any approach to constancy of E.M.F. was arrived at.

CHAPTER II.

BATTERIES.

11. Daniell Cell.—In the Daniell cell polarisation is overcome by surrounding the copper plate with a solution of sulphate of copper, so that the hydrogen at the moment of its development, instead of accumulating on the surface of the plate, attacks this copper sulphate, turning out the copper, which deposits on the plate as a metallic layer, and chemically takes its place, forming hydrogen-sulphate—*i.e.*, sulphuric acid. The plate is thus kept clean during the action of the cell by the addition of pure metal, while the sulphate of copper is consumed and the solution acidulated. Under these circumstances scarcely any polarisation (*see* para. 10) takes place, and the E.M.F. of the cell is nearly constant under every rate of current supply. It is convenient to remember that the volt (*see* para. 4) is approximately the E.M.F. of one Daniell cell. Even with care in the choice of pure materials and in the strengths of solutions, differences of as much as 2 per cent. will be found in the E.M F. of a series of cells, but the average under most favourable conditions is 1·08 volt each.

12. Local Action.—This cell's chief disadvantage lies in the fact that zinc, like hydrogen, decomposes copper sulphate, forming zinc sulphate and throwing out the copper. If, therefore, any copper sulphate reaches the zinc plate it is immediately converted into zinc sulphate, and the copper is thrown down upon the plate. Each separate particle of copper so deposited sets up, by contact with the zinc plate,

a separate E.M.F., by which a local current is established, flowing from the particle through the area of contact to the plate, and from the plate back to the particle through the surrounding solution, as shown in Fig. 2, which gives a magnified representation of such a particle, C, on the surface of the plate Z. This local current continues as long as the contact between Z and C is maintained, and the zinc in the vicinity of the particle is consumed, while the main current of the cell, receiving no accession through this source, is rather impeded by the obstacle C. This local current, decomposition, and wasted consumption of zinc is termed *local action*. It is

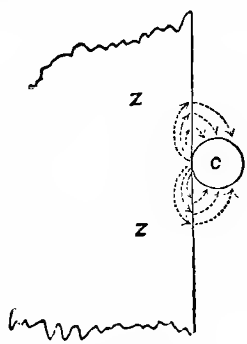


FIG. 2.

clear that any metallic impurity in the zinc plate, and at its surface of contact with the solution, tends to set up local action ;. and a plate of zinc containing such impurities consumes irregularly in action, and will show signs of pitting after merely remaining plunged in water for some time. Local action due to metallic impurity may be got rid of by cleansing the zinc plate and rubbing it well with mercury—amalgamating it, as it is termed. The superficial amalgam or alloy of zinc so formed has practically the same E.M.F. with copper as unprotected zinc, but the impurities are kept covered and separated from the solution by a metallic film of mercury, and no series of metallic contacts in a circuit from which liquids are excluded will give rise to an E.M.F. Impurities, however,

that are deposited from without—as, for example, particles of reduced copper from stray copper sulphate in the solution round the zinc plate—are not so readily imbedded in the mercuric film as to pass out of free contact with the solution, and so local action from this cause takes place in spite of amalgamation.

13. **Porous Partition.**—It would, of course, be possible to prevent the copper sulphate solution surrounding the copper plate from straying towards the zinc, by setting a partition or barrier of impervious substance between the plates, thus keeping them and their solutions entirely separate; but if this partition were of non-conducting material it would effectively prevent the current set up by the cell's E.M.F. from passing between the plates, and if, on the other hand, it were of conducting material, such as metal, then hydrogen would be thrown up on its surface, and polarisation would take place, so that the plan of introducing copper sulphate would be rendered useless. The difficulty is met by employing a partition of porous earthenware, a material in itself non-conducting, but containing numerous small channels, which become filled with liquid, so that, while the solutions on each side are kept apart, no great obstacle is interposed to the passage of the current. Cells which thus employ a separate solution for each plate are called double-fluid cells.

14. **Internal Resistance.**—All such porous partitions do, however, allow the liquid solutions to mingle to a greater or less extent, the process of so doing being termed *diffusion.* This diffusion takes place more slowly as the density of the separated solutions becomes more nearly equal, and as the porous barrier becomes thicker; but by increasing this thickness we add to the obstruction which is offered to the current in traversing the partition. This obstruction is termed *resistance*, and as electricity flows in virtue of E.M.F., so that flow is obstructed and diminished in rate in virtue of resistance. The flow of water presents an analogy, for if a current of water be flowing through a pipe from one tank to another owing to a difference of level at which the water stands in them, the

rate of flow will depend, assuming this difference of level artificially maintained, upon the dimensions of the connecting pipe. By altering the length and calibre of the pipe, or by altering, as with a tap, the amount of obstruction in it, we can make the current of water as great or as small as we please; in other words, we can make the quantity of water which passes through the pipe in a given time as great or as small as we desire. Just in the same way, having a certain E.M.F. in a circuit, we can, by increasing or decreasing the resistance in that circuit, make the current generated by the E.M.F. as great or as small as we please; or, in other words, we can alter at will the quantity of electricity which passes through the circuit in a given time. A good conductor, offering little obstruction to the passage of an electric current, has little resistance; while a bad conductor is one that has, on the contrary, great resistance. We shall consider the nature of resistance more fully at a later period, but it will at present suffice to observe that it is measurable in terms of a definite unit, called the *ohm*, after the German scientist of that name. An ohm is roughly the resistance offered by a column of mercury one metre long and one square millimetre in section, or of 120 yards of ordinary No. 8 overhead iron telegraph wire, or of 200 yards of ordinary cable core, whose conductor weighs 120lb. per nautical mile; in other words, each of these three lengths of conductor would offer approximately the unit amount of obstruction to the flow of electricity through them.

15. Types of Daniell.—We have seen that the Daniell cell, by the diffusion of its solution, infringes the second condition of para. 5, page 3, by the waste of material which goes on when the cell is not at work, but it has otherwise so many advantages that various forms have been adopted for different purposes and conditions of current supply.

16. Gravity Daniell.—One of the simplest types is the gravity Daniell, shown in Fig. 3. The liquids are kept apart by gravitation only, the solution of copper sulphate being denser than that of zinc sulphate. A flat plate of copper, C,

is placed on the floor of the cell as the negative element, and covered with crystals and solution of sulphate of copper up to L, about one-fourth of the cell's height. The zinc solution of acidulated water or sulphate of zinc is then carefully added, and the zinc plate Z supported last of all a little below the surface H. Diffusion of the solutions proceeds rapidly if the cell is out of action, but sustained work tends to keep the diffusion in check owing to the more active consumption during current supply. The cell is therefore well suited for yielding continuous currents, and has always been much in favour for telegraphic purposes in the United States, where the circuits are worked on the closed-circuit principle—that is to say, with the current from the battery always flowing to line, except in the spaces or pauses of manipulation separating the signals.

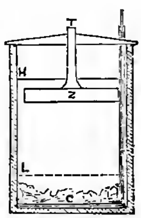


FIG. 3.

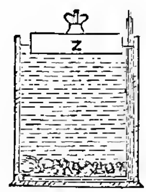


FIG. 4.

17. Minotto Cell.—The Minotto cell, shown in Fig. 4, is a derivative of the gravity type, more especially adapted to transport and discontinuous work. The outer cell is commonly of gutta percha, which is a substance less liable to injury than glass. Above the copper plate and sulphate crystals wet sawdust is packed, and the zinc plate rests above the sawdust with the pressure of its own weight, and with its upper surface about level with the top of the cell. Whereas the gravity cell cannot be moved without disturbing the solutions and assisting their diffusion, the Minotto suffers little by transport. The sawdust takes the place, in fact, of the porous earthenware partition in the typical Daniell cell. The internal resistance is, however, greater, varying from 10 to 30 ohms per cell, according to the quality and packing of the sawdust. If the

cell is first set up with sawdust soaked in fresh water, the internal resistance (*see* para. 14) will be 100 ohms or more, until by working the zinc sulphate and sulphuric acid have time to form and impregnate the sawdust. The cells, if made up in this way, should, therefore, be left on *short circuit, i.e.*, with the poles (*see* para. 8) directly connected for twenty-four hours or so before they are brought into use. If required to be set up for more immediate use, the sawdust may be soaked in acidulated water. They tend to increase in internal resistance after a time, owing to the drying up of the sawdust, and fresh water has to be added when necessary, the right condition of moisture existing when the sawdust is thoroughly saturated, and a layer of liquid just rises round the base of the zinc.

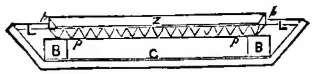


FIG. 5.

18. **Tray Cell.**—The Thomson tray cell is another form of Daniell, of the gravity type, adapted for supplying strong and continuous currents. Shallow trays, 22in. square and 2¾in. deep, form the outer cell. A square sheet of copper rests on the floor of the tray, and the sulphate of copper in crystal and solution rises above to a height of some 2in. The zinc plate, in the form of a stout grating 16in. square, shown in section at Z in Fig. 5, is supported horizontally on stoneware blocks, B B, at the corners of the tray. Stout parchment paper, *p p*, covers the lower surface and sides of the zinc grating to asists in separating the solutions, on the principle of a porous diaphragm, and the zinc solution—fresh water, or, preferably, a solution of sulphate of zinc—is poured in above until the upper surface of the grating is covered to the depth of about a quarter of an inch. The resistance of these cells is about one-tenth of an ohm each, or even less when in good working order. They require charging with sulphate of copper at regular intervals, and the

density of the zinc sulphate solution should also be kept down to a sp. gravity of 1·12, or about one-quarter saturation, by partly drawing off from time to time, and replacing with fresh water.

19. **Leclanche Cell.**—A type of cell much in use for discontinuous work is the Leclanché. This is a single-fluid cell in which zinc is the positive element (*see* para. 8), carbon the negative, and a solution of salammoniac the exciting liquid. The carbon plate is surrounded by crushed carbon in intimate admixture with black oxide of manganese, either by being inserted into a porous jar with those materials, or, in what is

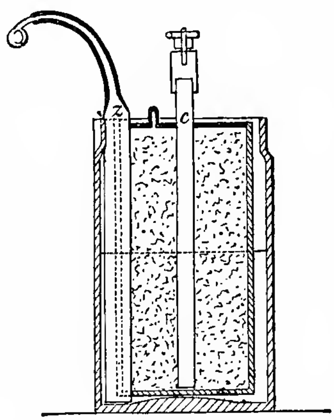


FIG. 6.

called the agglomerate cell, by having those materials solidly cemented round the carbon plate in a cylindrical form under pressure. The zinc element is generally in the form of a rod, placed in one corner of the cell, which is represented in section at Fig. 6. During the cell's action the salammoniac, which is a compound of ammonia and chlorine, is decomposed, the chlorine attacks the zinc, forming zinc chloride, which, being soluble, disappears into the solution, while the ammonia developing at the surface of the carbon also forms a soluble compound with oxygen, which it seizes from the oxide of manganese. Thus, in action, the zinc is con-

sumed, its chloride accumulates in solution, the carbon is unaltered, and the oxide of manganese loses some of its oxygen. Practically, however, when the supply of current from the cell exceeds a certain rate, and consequently when the chemical decomposition reaches a certain degree of activity, the above simple set of reactions is not strictly carried out; secondary and more complex chemical products form, which soon culminate in the development of hydrogen at the surface of the carbon plate, and polarisation (*see* para. 10) sets in, unless resting time is allowed for the secondary products to dissolve. For this reason the Leclanché cell is not suited for strong and continuous current supply, and its E.M.F. (*see* para. 4), normally about 1·48 volts, soon falls under hard work. For more limited, and especially for intermittent current supply, however, the cell has many advantages. Its internal resistance is generally between 2 and 5 ohms, and can be made much lower if, with this object, the cell be heated. It requires very little supervision, and when engaged on light work, such as the occasional ringing of bells, will not unusually continue to operate for two years without any attention or recharge. If the zinc rods be of good material, there is also exceedingly little local action.

20. **Fuller's.**—Fuller's bichromate cell is a double-fluid form of the zinc-carbon combination. The outer cell is a stoneware jar, as shown in section at Fig. 7. In it stands the carbon plate C, and the porous pot *p p*, filled with water to the level L, holding the zinc plate Z, which is usually of conical form, and kept permanently amalgamated by dipping into mercury placed for that purpose at the bottom of the porous cell. The solution for the outer jar is of bichromate of potash in water acidulated with one-ninth of its volume of sulphuric acid. During hard work polarisation takes place to some but not as a rule to any serious extent, and the E.M.F. of the cell is approximately 2 volts, and its internal resistance from about 2 to 5 ohms.

21. **De la Rue's.**—A very convenient form of cell for testing and light signalling work is De la Rue's single-fluid chloride of silver cell, shown in section at Fig. 8. The outer cell

is a glass tube one inch in diameter, with a flat bottom. It contains a solution of 200 grains of salammoniac to the pint of water, and the elements are zinc and silver. The latter is in the form of a wire, *w*, fused into a cylindrical mass of chloride of silver, which is further protected by a paper tube, *p p*, to keep it from coming into contact with the zinc rod, *z z*. If set up with care, and not subjected to very hard work, the E.M.F. of this cell is remarkably constant at about 1·03 volts, and the average is so well maintained by different cells that in the absence of a standard of E.M.F. more accurate, it may be

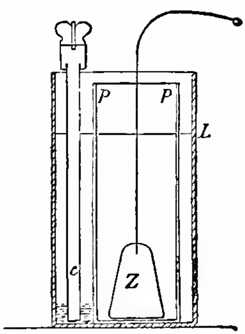


FIG. 7.

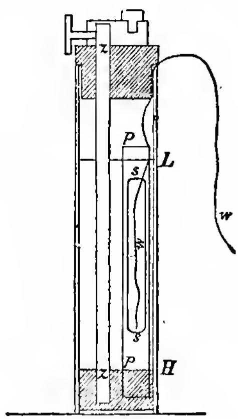


FIG. 8.

generally relied upon for that purpose to a limit of 1 per cent. when comparing measurements made at different places. Its internal resistance in good order is from $1\frac{1}{2}$ to 5 ohms. The dimensions of the plates prevent the use of these cells for hard work, as the zinc would soon be entirely consumed; but they take up so little space, are so easily insulated, and, once carefully set up, require so little attention, that they are admirably adapted for testing and signalling, particularly on board ship. As the zinc should never touch the paper tube in the cell, it is a good plan when setting up the latter to first pour in melted paraffin wax to the height (H, Fig. 8) of about

half an inch, and then to lower the elements into their proper opposite positions, so that they remain fixed at their lower ends as the wax solidifies. It is also well, in order to prevent corrosion, to whip with silk the external loop of silver wire which connects cell with cell.

22. **Standard Cell.**—A cell which is most useful in testing, but used almost solely for the purpose of supplying a reliable standard of E.M.F., is the Clark standard cell. This is a combination of pure zinc and mercury, and its E.M.F. is 1·45 volts at 66° F., diminishing very slightly as the temperature rises. This E.M.F. is very constant, and the amount of its variation between different cells is generally exceedingly small. To maintain its constancy this cell should never be allowed to send a strong current. Its internal resistance is therefore immaterial; but as it tends ultimately to increase enormously with age through the drying up of the exciting salts, it is necessary to ascertain when employing the cell that its resistance is not excessive.

CHAPTER III.

ELECTROMOTIVE FORCE AND POTENTIAL.

23. **Cells in Series.**—The usual mode of connecting cells together to unite their powers is by joining the positive pole of one cell to the negative of the next all through the series. The E.M.F. and resistance between the terminal poles of the battery so formed will be the sum of the E.M.F.'s and resistances of the individual cells. Thus, if the zinc pole of a Fuller cell, whose E.M.F. is 2 volts and resistance is 3 ohms, be connected to the carbon pole of a Leclanché, whose E.M.F. is 1·5 volt and resistance is 5 ohms, then between the carbon pole of the Fuller and the zinc pole of the Leclanché respectively—the positive and negative poles of the two-celled battery so formed—there will be an E.M.F. of 3·5 volts and a resistance of 8 ohms. Similar reasoning applies to any number of cells. So that if we connect up *in series*, as it is termed, 20 Daniells, each having an E.M.F. of 1·05 volts and a resistance of 10 ohms, we shall obtain a battery of 21 volts E.M.F. and 200 ohms total internal resistance.

24. **Leakage.**—It is important to remember that the necessity for the insulation of cells increases with the total E.M.F. accumulated, or, in other words, with the number of cells connected in series. For although a slight film of moisture on the outer surface of a glass cell may only allow an inappreciably small current to pass from pole to pole of that cell singly, the loss of current may become serious when that leakage may form the path of escape to a large E.M.F.; in the same way that a small leak at the base of a tank may cause

only a trifling escape of water when the tank is nearly empty, but might allow of serious loss as the level of water and consequently the pressure is raised from within. It should be the object, therefore, with large batteries, especially when they are employed in delicate measurements, to place them on insulating supports, and also to prevent leakage from cell to cell by keeping the outer surfaces of the cells clean and dry. A good method is to coat with paraffin wax the outer surfaces of the cells and the trays or brackets on which they stand.

25. **Potential.**—While the poles of any cell are insulated their potentials will be equal and of opposite signs. For example, if an insulated Daniell cell has an E.M.F. of 1·08 volt, the potential of its copper or positive pole will be 0·54 +, or positive, and that of its zinc or negative pole 0·54 −, or negative, the difference of potential between them being thus 1·08, equal by definition to the E.M.F. Similarly with an insulated battery of 100 volts E.M.F., the potential of the positive pole will be + 50, that of the negative pole − 50. Consequently, if the battery be homogeneous, the potential at its centre must be zero. This zero potential is really the potential of the earth and of all neighbouring conductors in connection with it. So that, like the effect of the moon's gravitation on the ocean, which produces in the tides a fluctuation of level above and below the normal level of the sea, the effect of the contact of the different metals of a cell, either singly or as accumulated in a battery, is to produce at the insulated poles potentials equally deviating above and below from the potential of the earth, which, being constant, is reckoned as zero. So that when we speak of potential we tacitly refer to this earth potential zero, but no such allusion is necessarily made when we discuss difference of potential, viz., E.M.F.; just in the same way, when we speak of the level of a position as being 3,000ft. high, we refer to the mean level of the sea, while difference of elevation is an idea in which the sea level may have no part.

26. **The Distribution of Potential in a Battery.**—Fig. 9 shows how we may graphically represent the rise of potential in an insulated battery. The thick lines similar

to *zz* across the dotted line A B represent the zinc plates, and the thinner lines similar to *c c* the copper or carbon plates of the battery. Now, holding up the paper so that its plane is vertical, the upright lines similar to A D represent in length and direction the potential in the battery at the points on which they rest. Positive potentials are represented by lines above A B, negative potentials by lines below. Suppose we take a scale of measures such that a difference of potential of one volt shall be represented by one-eighth of an inch. As we may select any horizontal scale we please, it will be con-

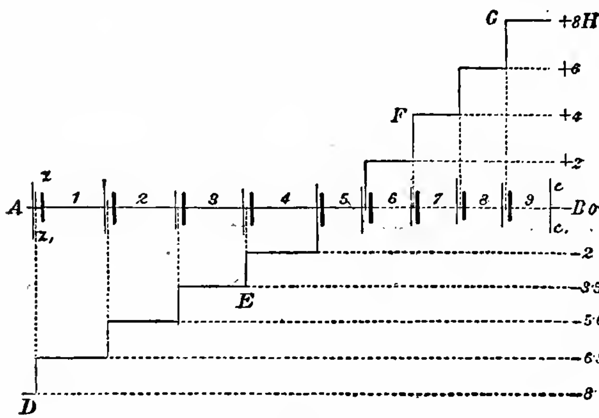


Fig. 9.

venient to make it correspond to the internal resistance of the battery, and our diagram will then embody both E.M.F. and resistance. Let us take a horizontal scale along the line A B of 8 ohms to the inch. Now let us consider the case of an insulated battery, A B, of nine cells, numbered as shown, the first four being Leclanché's, each of 1·5 volt E.M.F. and 3 ohms resistance, and the last five Fuller's, each of 2 volts E.M.F. and 2 ohms resistance. Then the total E.M.F. will be 16 volts and resistance 22 ohms, so that the latter will be represented on our scale of 8 ohms to the inch by placing the poles of the battery $\frac{22}{8} = 2\frac{3}{4}$ inches apart, so that the line

A B must be $2\frac{3}{4}''$ long. Each cell may now be marked off along this line, the distance from one connection to the next being $\frac{3}{8}''$ for the Leclanchés and $\frac{1}{4}''$ for the Fullers. The plates may then be sketched in their right positions, as shown. The vertical scale has now to be considered. We have seen in the preceding paragraph that the points A and B will receive equal and opposite potentials, each equal to half the total E.M.F., which in this case is 16 volts ; consequently, B being the positive pole will have a potential of + 8, represented on our scale by a point one inch above A B, while the negative pole

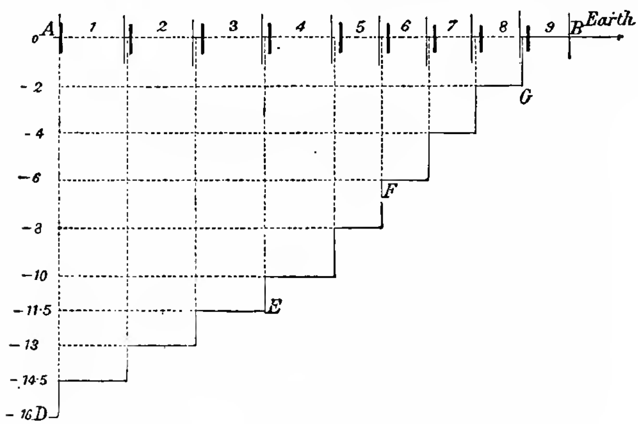


Fig. 10.

A will have a potential of − 8, indicated by the point D one inch below A. By then carrying horizontal lines from D and H parallel to A B, and altering the potential opposite each junction of plates by the exact amount of E.M.F. that junction generates to scale, we shall produce the zigzag line D E F G H, every point of which represents by its distance from A B the potential of the point opposite to it in the battery.

27. **Zero Potential in a Battery.**—In the particular case of the battery we have been dealing with, we see from the figure that the potential line in the first Fuller cell, No. 5, coincides with the line A B, and since A B is also,

as explained in para. 25, the earth potential or zero line, the potential of cell No. 5 must be zero. Consequently, the potential line D E F G H will not be disturbed if this cell be connected with the earth by dipping a wire into it whose other end is deeply buried in the ground, for example; for by so doing we simply connect two bodies at the same potential, and no current tends to flow under ordinary circumstances between two bodies at the same potential any more than water tends by gravitation to flow between two points at the same level.

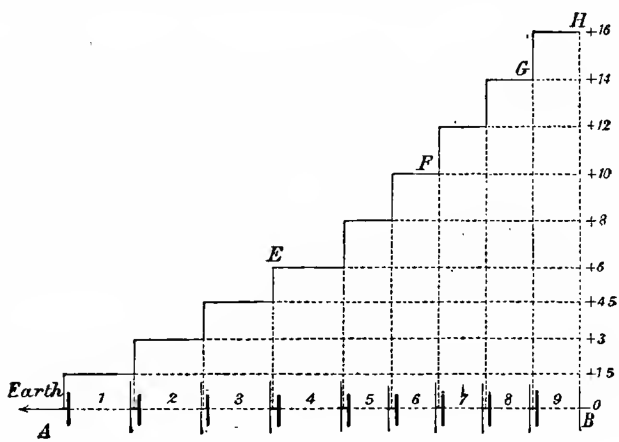


FIG. 11.

28. **Earth Connections.**—If, however, the earth wire be removed from No. 5, and connected to any other point, the position of the line of potentials will be altered. For example, if the earth wire be connected to B, the positive pole, as in Fig. 10, a current will tend to flow from B to earth, but cannot be maintained because it can find no circuit, the battery being otherwise insulated. The point B and the earth cannot, however, retain different potentials after connection, so B's potential falls to zero. But the mere connection of B with earth cannnot alter the E.M.F. of the battery—that is to say,

cannot alter the difference of potential existing between the poles A and B; so that as B has lost a potential of 8, A must be augmented by that amount, and will therefore become -16, as shown, and, filling in the sections of the potential line as before, we find that it has descended bodily from its previous position through the distance H B. Similarly, if B be insulated and A instead earthed, as in Fig. 11, the potential of A becomes zero, that of B makes up for the loss by becoming $+16$, and, all other points retaining their relative potentials to these, the potential line is moved bodily up to the position shown.

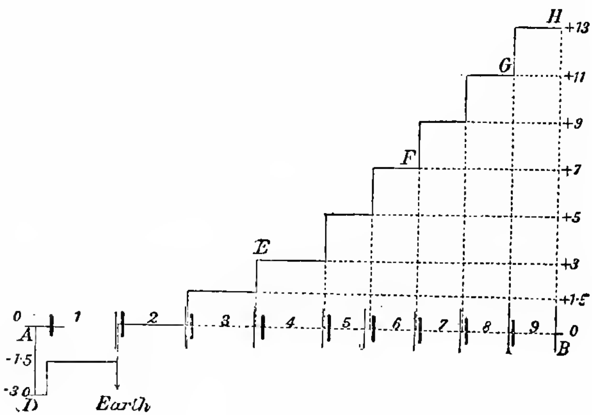


FIG. 12.

Again, Fig. 12 shows the effect of putting to earth a particular point in the battery otherwise insulated. We thus see that in all cases of connecting to earth, the potential of the point earthed is brought to zero, and the potentials of all the remaining points in the battery follow from this point by the simple summation of the individual E.M.F.'s as they are met with.

CHAPTER IV.

RESISTANCE.

29. Relative Resistance of Metals.—We have already seen (*see* para. 14) that the resistance of any electric path is the obstruction which a current traversing it encounters, and we are now in a position to examine the nature of this obstruction more closely.

Of all known substances metals offer the lowest resistances, and the more common of these may again be comparatively classed in the following list, which commences with the least resisting and ends with the most resisting metal:—

Relative Specific Conductivity.		Relative Specific Resistance.
1·000	Silver (hard drawn)	1·000
0·996	Copper „ „	1·005
0·780	Gold „ „	1·283
0·290	Zinc (pressed)	3·446
0·168	Iron (annealed)	5·949
0·131	Nickel „	7·628
0·124	Tin (pressed)	8·091
0·083	Lead „	12·131
0·046	Antimony „	21·645
0·016	Mercury (liquid)	62·500
0·0125	Bismuth (pressed)	80·000

30. Relative Specific Resistance and Conductivity.—Since in this list, taken from Clark and Sabine's tables, silver is the metal offering least resistance to a traversing current, it must also be the metal which most facilitates its passage—that is to say, which conducts it best. Silver is therefore said to

be the metal of greatest *conductivity.* The table therefore gives the metals in diminishing order of resistance by ascent, and in diminishing order of conductivity by descent. The figures in the column on the right hand represent the resistance of each metal relatively to that of silver taken as unity, or, as it is called, the *relative specific resistance*; while the figures in the left hand column give the conductivity of each metal relatively also to that of silver as standard, or, as it is termed, the *relative specific conductivity.* Thus iron is shown to have a resistance of nearly six times that of silver, so that if a conductor made of pure hard-drawn silver were of such dimensions as to offer a resistance of one ohm, a conductor of annealed iron of exactly the same dimensions would offer about six ohms resistance. Consequently, since iron resists a current six times as much as silver, the conductivity of silver must be six times that of iron; therefore, calling the conductivity of silver 1, that of iron must be six times less, or one divided by six—that is, $\frac{1}{6}$, or 0·167, the more exact number being, by the table, $0·168 = \frac{1}{5·949}$. Similarly, the relative specific resistance of mercury being 62·5, its relative specific conductivity is $\frac{1}{62·5}$, or 0·016—that is to say, 1·6 per cent. of pure silver; so that in all cases the specific conductivity of any material relatively to a standard material is the quotient found by dividing into unity the specific resistance of the substance relatively to the same material; and, further, the conductivity of any path is found from the resistance of that path in the same manner and is expressed in terms of a unit, to which the name of *mho* has been given by Sir William Thomson, the term conveying the inverse idea of resistance by being an inversion of the word "ohm." Thus, if a certain conductor had a resistance of 0·2, 1, 50, or 7,000 ohms, its conductivity would be $\frac{1}{0·2}$, $\frac{1}{1}$, $\frac{1}{50}$, or $\frac{1}{7,000}$ mhos respectively—that is, 5, 1, 0·02, or 0·000143 mhos.

31. Decrease of Conductivity with Impure Metals and Alloys.—The relative specific conductivity of metals is always reduced by impurity, and even a very slight degree of impurity will suffice to palpably lower the conductivity, or, in other words, add to the conductor's resistance. It is by no means impossible to find a sample of impure commercial copper which will only possess the specific conductivity of zinc, owing to the admixture of foreign and non-conducting material, or of baser and more resisting metal. Also a conductor composed of an alloy of different pure metals in known proportions has generally a greater specific resistance than would be furnished by an estimate made on the supposition of its being a bundle of separate and purely metallic conductors of proportional dimensions.

32. The Use of Copper for Conductors.—The table also shows that pure copper is so close to pure silver in conductivity as to scarcely make it desirable to select the latter and rarer metal for conductors, even were the matter of cost entirely set aside. In telegraphy, therefore, copper is the metal employed as the conductor for submarine and subterranean purposes, while both iron and copper are used for overhead wires.

33. Resistance of a Wire at Constant Temperature Not Affected by Strength of Current.—The resistance of a conductor of homogeneous material depends only upon its dimensions and temperature. So that if the temperature of a fixed conductor be constant, its resistance will be the same for every strength of current.

34. Resistance of a Given Wire Proportional to its Length.—As regards dimensions, the resistance of a conductor of homogeneous material depends upon its cross-section and length. If its cross-section be constant, the resistance will be proportional to the length. Thus, if a mile of a certain wire had a resistance of 10 ohms, then the resistance of 0·02, 0·4, 7, or 5,000 miles of the same material and cross-section, would be 0·2, 4, 70, or 50,000 ohms respectively.

35. Resistance of a Given Length of Wire Inversely Proportional to its Cross Section.—With conductors of the same length and material, the resistance increases as the cross-section is reduced, just as the resistance to the flow of water through a pipe is increased as the pipe is reduced in cross-section. Thus, if two wires, A and B, are of the same length and material, but B has twice the cross-sectional area of A, then B may be regarded as being made up of two wires very close together, each of A's cross-section, and therefore equal to A in every respect. Obviously the double wire would convey under the same E.M.F. double the current that would traverse the single wire A, and its conductivity being doubled, its resistance would be only half that of A. Similarly, another wire, C, of the same length and material, but of three times A's sectional area, would offer one-third of A's resistance; and in the same way, if C had 0·25, 0·7, 3, or 15 times the cross sectional area of A, it would offer $\frac{1}{0{\cdot}25}$, $\frac{1}{0{\cdot}7}$, $\frac{1}{3}$, or $\frac{1}{15}$; that is, 4, 1·43, 0·33, or 0·061 times A's resistance respectively.

36. Conductivity of a Given Length of Wire Proportional to its Weight.—Further, since in doubling the sectional area of a given length of homogeneous wire we double the quantity of metal in it, we necessarily double the weight of the wire, so in every case the conductivity of a wire of fixed length and material will be proportional to its weight. Thus, if a wire, A, one mile long, weighing 100lb., offers 10 ohms resistance, then another wire, B, of the same material, one mile long, and weighing 200lb., would have double the cross-sectional area, double the conductivity, and consequently half the resistance of A, namely, 5 ohms; and in the same way, if B were to weigh 0·5lb., 2lb., or 500lb., it would offer $\frac{100}{0{\cdot}5}$, $\frac{100}{2}$, or $\frac{100}{500}$ times; that is, 200, 50, or 0·2 times the resistance of A respectively.

37. Relation Between Length, Diameter, and Weight of Telegraph Conductors.—The conductors employed in telegraphy

are wires—that is to say, are cylindrical in form, so that their dimensions are generally—

(1) **Length** (on land in statute miles of 1,760 yards, at sea in nautical miles of 2,029 yards), and

(2) Either the **Diameter** or the **Weight** per mile.

If the diameter is given, the sectional area is found by multiplying the square of the diameter by 0·7854. For example, a wire 0·03in. in diameter would have a sectional area of $0·03 \times 0·03 \times 0·7854$, or 0·00071 square inch. The sectional area and weight of a wire are directly connected, for in the case of iron wire the sectional area in square inches multiplied by 17,500 gives approximately the weight in pounds per statute mile. For copper wire the similar multiplier is 20,000, but for cable core, whose conductor is usually a strand of seven copper wires, the approximate weight of conductor per nautical mile is found by multiplying the sectional area found from the diameter as above by 18,100. For example, a copper strand of cable core has a diameter of 0·13in. Its area will therefore be approximately $0·13 \times 0·13 \times 0·7854$, or 0·01327 square inch, and 0·01327 multiplied by 18,100 gives 240·2, the approximate weight in pounds of one nautical mile of this strand.

38. Practical Problems in Telegraph Conductors. — The problems which practically arise for solution in the case of telegraphic conductors are generally of the following form :—Having given the metal of which a wire is made, and any two of the three following particulars—dimensions, resistance, and conductivity relatively to pure metal, it is required to find the third. Leaving the effect of temperature at present out of consideration—that is to say, supposing that the temperature remains constant—we shall find no difficulty in solving any such problem, as the following cases will show :—

(*a*) Having given the dimensions and resistance, to find the conductivity.

Example.—A copper strand conductor of cable core 5 knots long and 0·096in. in diameter gives a resistance at the

standard temperature of 48·42 ohms. What is its conductivity compared with pure copper?

The sectional area of the strand is $0·096 \times 0·096 \times 0·7854$, or 0·007238 square inch approximately. Its weight per knot is therefore $0·007238 \times 18{,}100$, or 131lb. As may be seen in electrical tables, the resistance of a wire of pure copper one knot (nautical mile) long, and weighing altogether 1lb., would be 1196·7 ohms at the standard temperature of 75° F.; consequently, the resistance of a wire of pure copper one knot long and weighing 131lb. would be (*see* para. 36) at the standard temperature $\frac{1196·7}{131}$, or 9·135 ohms, and hence its conductivity would be $\frac{1}{9·135}$ mho—that is, 0·10947 mho. The wire in question offering 48·42 ohms, is five knots long, so that one knot would offer $\frac{48·42}{5}$, or 9·684 ohms, and the conductivity of one knot would therefore be $\frac{1}{9·684}$, or 0·10326 mho. The conductivity of pure copper is to the conductivity of this wire, therefore, in the proportion of 0·10947 mho to 0·10326 mho—that is, as 1 is to 0·9433, or as 100 is to 94·33; so that the wire's conductivity is thus $94\frac{1}{3}$ per cent. that of pure copper.

(*b*) Having given the dimensions and conductivity, to find the resistance.

Example.—A copper wire 600 yards long and 0·25in. in diameter has a conductivity 96 per cent. that of pure copper. What will be its resistance at the standard temperature?

The sectional area is in this case of a cylindrical wire, not a strand, strictly $0·25 \times 0·25 \times 0·7854$, or 0·0490 square inch, so that its weight per statute mile will be $0·049 \times 20{,}000$, or 980lb. approximately. The resistance of a statute mile of pure copper weighing one pound will be found in electrical tables to be 872 ohms at the standard temperature, which for overhead land lines is generally taken at 60° F. A wire of pure copper weighing 980lb. per mile would therefore have a

resistance of $\frac{872}{980}$, or 0·8898 ohm, and its conductivity will thus be $\frac{1}{0·8898}$, that is, 1·1238 mho; but its conductivity, being only 96 per cent. of pure copper, will be 96 per cent. of 1·1238, or $0·96 \times 1·1238$—that is, 1·0789 mho, or $\frac{1}{1·0789}$, namely, 0·9268 ohm per mile. But the wire in question is only 600 yards long, and will therefore have less than this resistance in the ratio of 600 to 1,760, so that its resistance will finally be $\frac{600 \times 0·9268}{1760}$, or 0·316 ohm.

(*c*) Having given the resistance and conductivity, to find the dimensions.

Example.—A length of homogeneous cable strand conductor, 0·133in. in diameter, is known to be of 97 per cent. conductivity compared with pure copper, and has a resistance at the standard temperature (75° F.) of 37·5 ohms. What is the length of the conductor?

The area of this wire is approximately $0·133 \times 0·133 \times 0·7854$, or 0·013893 square inch. Its weight per knot is therefore $0·013893 \times 18{,}100$, or 251·5lb. The resistance of a wire of pure copper one knot in length and of this weight would be $\frac{1196·7}{251·5}$, or 4,759 ohms at the standard temperature. Its conductivity would therefore be $\frac{1}{4·759}$, or 0·21013 mho. The wire in question, however, having 97 per cent. of this conductivity, would have only $0·21013 \times \frac{97}{100}$, or 0·20382, and therefore a resistance of $\frac{1}{0·20382}$, or 3·988 ohms, but its total resistance being 37·5 ohms, its length must be greater than one knot in the proportion of 37·5 to 3·988, so that its length will be $\frac{37·5}{3·988}$, or 9·425 knots.

The steps in all these calculations can of course be very much abbreviated when the method is clearly understood. Thus, the example in (*b*) is capable of being worked out directly by one fraction in the following form :—

$$\frac{872 \times 100 \times 600}{0{\cdot}25 \times 0{\cdot}25 \times 0{\cdot}7854 \times 20000 \times 96 \times 1760}$$

that is, 0·316.

39. Multiple and Sub-Multiple Units of the Ohm.—Since the resistances which are dealt with in telegraphy may sometimes be exceedingly small (perhaps that, for instance, of a short length of thick copper wire offering a minute fraction of an ohm), and on the other hand may be very great, (such as that of the insulating gutta-percha covering of a short length of cable core amounting, it may be, to billions of ohms), it is convenient to adopt two secondary units, one a multiple and the other a sub-multiple of the ohm. Thus a million ohms is spoken of as a megohm, and is usually represented for brevity by the last capital letter, omega, of the Greek alphabet (Ω)—thus, 227·5Ω means 227,500,000 ohms. The ohm itself is commonly represented by the small letter omega (ω). One-millionth of an ohm is called a microhm, and has at present no representative symbol. Thus, 7,560 microhms is the same as 0·00756ω.

40. Resistances in Series.—Resistances that are connected in series or simple circuit are always directly additive. This is obvious from the fact that any resistance may be regarded as an equivalent length of some particular wire, and we have seen that the resistance of a wire is proportional to its length. Thus, if a circuit be composed of a battery of 40ω total internal resistance, an overhead conducting wire of 50ω, and the copper wire wound on the coils of a sounder of 35ω, then we may regard that circuit as being equivalent to 135 miles of wire whose resistance is 1 ohm per mile, the resistance of forty such miles being in the battery, fifty in the line, and thirty-five in the receiving sounder.

41. Resistances in Multiple Arc.—Resistances that are not connected in series, but side by side, or as it is termed "*in multiple arc*," cannot of course be additive, for between the points they connect there is more than one path, and the conductivity is increased by each additional path so introduced. We therefore have in this case the conductivities additive. Suppose A B C and A D C to be two wires connected at A and C, and each offering 5ω, then the conductivities of these wires are each $\frac{1}{5}$ or 0·2 mho. The double conductor may be regarded as a single conductor of double sectional area, so that we know its conductivity will

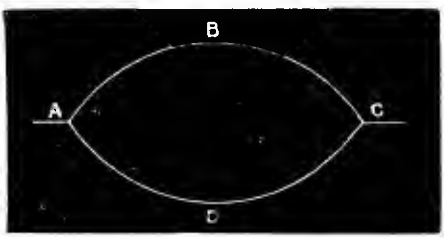


Fig. 13.

be doubled—that is, 0·4 mho—and the resistance of the two together, or, as it is termed, their *joint resistance*, will be $\frac{1}{\cdot 4}$ or 2·5ω, half that of either branch. So that the joint resistance of two equal wires is half that of one, of three equal wires one-third that of one, of ten one-tenth, and so on. When, however, the wires are not equal, the case, though not so simple, is similar. Thus if in Fig. 13 A B C had 10ω, while A D C had 20ω, then the conductivity of A B C would be $\frac{1}{10}$ or 0·1 mho, that of A D C $\frac{1}{20}$ or 0·05 mho. The joint conductivity is their sum, 0·1 + 0·05, or 0·15 mho—that is, $\frac{1}{0 \cdot 15}$ or 6·667ω, and similarly with any number of

parallel conductors. For example, in Fig. 14 five resistances are shown in multiple arc, of 150ω, 60ω, 55ω, 2,000ω, and 80ω respectively; their conductivities will therefore be

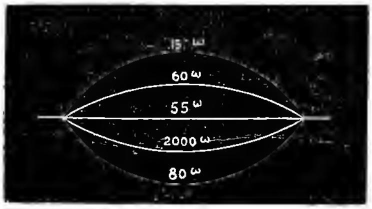


FIG. 14.

$\frac{1}{150}$, $\frac{1}{60}$, $\frac{1}{55}$, $\frac{1}{2000}$, $\frac{1}{80}$ mhos respectively—that is, 0·006667, 0·016667, 0·018182, 0·000500 and 0·012500 mhos. The joint conductivity will be the sum of all these, namely, 0·054516 mhos, and the joint resistance will be $\frac{1}{0·054516}$, or 18·343ω.

CHAPTER V.

CURRENT.

42. Effect of "Opening" or "Closing" a Circuit.—Having examined the nature of electromotive force as supplied by batteries, and of resistance, we are now in a position to deal with the laws which relate to the current which flows in any given circuit, laws which are among the simplest yet most important considerations of the subject.

Reverting to our typical form of simple cell at Fig. 1, page 4, we saw that no current flowed in the circuit B E F G H if the wire F G H were severed at any point. When a circuit is interrupted at any point it is said to be *opened* or *broken*, and when re-established to be *completed* or *closed*. Every possible circuit, even though it may embrace, as in telegraphy, hundreds of miles of conductor, and the earth itself between that conductor's terminals, may be regarded as a simple extension of that wire F G H, and hence a current in a circuit ceases and recommences upon the complete opening and closing of that circuit at any point, even though in the battery itself, as, for example, by withdrawing and replacing a battery plate in its cell.

43. Telegraph Signalling on the American Closed Circuit System.—Fig. 15 represents the simplest form of telegraphic circuit. A, B, and C are supposed to be three stations connected by the overhead conductors A B and B C. At each station there is a key, K, a sounder, S, and a switch, *s*. At A there is a battery, F, one pole of which goes to the key and the other is

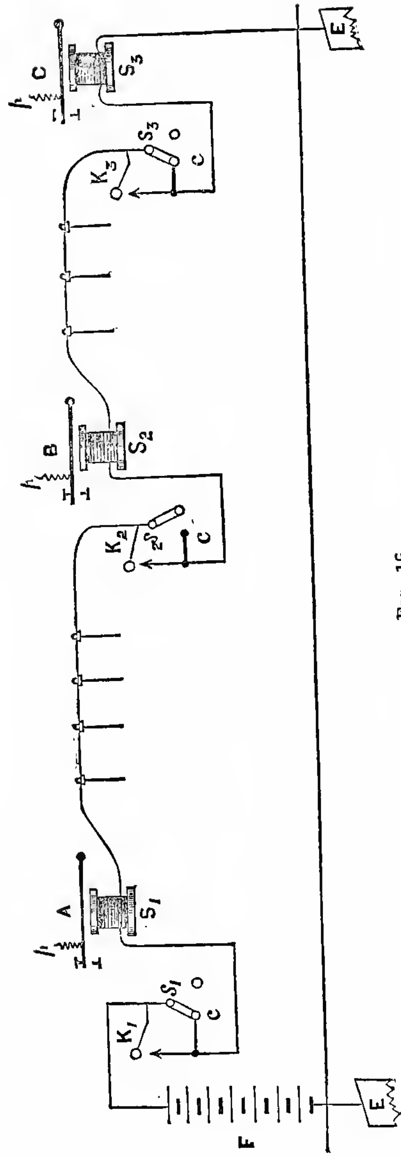


Fig. 15.

connected to the earth, say by a large metal plate buried in the ground. A similar earth connection is made at C. The switches have each two positions: when turned to the right, as at B, they break connection between the key terminals; and when turned to the left they re-establish it by making contact at *c*. The keys remain up while at rest through the action of a spring. If, now, all the switches are turned to the left or closed, a complete circuit is formed from the battery through $s_1 S_1$ A B, $s_2 S_2$ B C, $s_3 S_3$ and the earth, and a current will therefore flow through all the three sounders. The action of the current on the sounder we shall not at present consider, but it will suffice to observe that when the current passes through the wire of its coils, those coils attract the soft iron cross-piece or *armature* on the beam above, and pull it down against the attraction of the spiral spring *p*. If, now, B. opens his switch by turning it to the right the circuit is completely interrupted at his key, the current ceases, and the spiral springs reassert their control over the armatures, which rise to their upper limiting stops. If, now, B. depresses his key for a single moment and releases it again, he will have closed the circuit during that moment at the key contact, the current will have passed during that interval, and caused the armatures of all three sounders to be attracted down to their lower stops, only to recoil again when the key K_2 rises; so that whatever contacts B. makes, whether long or short, at his key, will be faithfully reproduced on all three sounders, whose armature levers follow, in fact, all the up and down motions of B's key. When B. has finished sending he closes his switch, and either A. or C. can open their switches and reply to him in the same way. Thus, with one battery, F, any convenient number of stations can communicate with each other along a line of conductor by inserting a key, switch and sounder. This system is called the closed-circuit system, already alluded to in para. 16. It is in almost universal use in the United States, but is not so suitable for the conditions of European telegraphy, and is scarcely employed at all in England, one great disadvantage being the waste of battery power while

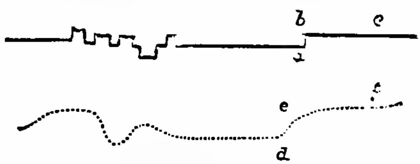
the circuit is silent, as the current is normally always flowing.

44. Retardation of Current Slight on Overhead Lines except at High Signalling Speeds.—The instant such a circuit as that in Fig. 15 is closed a current commences to flow, but it does not instantaneously arrive at its full ultimate strength. A certain amount of time is required for the current to rise to its constant maximum limit, and this period of time is called the *variable period* of the current's flow. The duration of the variable period depends not only upon the nature of the circuit itself, but also upon the nature of the space surrounding it; other things being equal, however, it increases with the length of the conductor. Practically, however, the variable period is in the case of overhead land lines exceedingly short. For example, with an ordinary No. 8 overhead wire 100 miles in length, with a battery and earth at one end and earth at the other—that is to say, with no telegraph instruments—the current at the end distant from the battery would arrive at 99 per cent. of its ultimate maximum value in 0·0021 second, or about $\frac{1}{500}$th part of a second, so that although the duration of the variable period is increased by the introduction of receiving apparatus into the circuit, still that period is, with overhead lines, so short that it may be neglected, except on lines worked at a very high signalling speed.

45. Retardation Considerable on Submarine and Subterranean Lines.—With submarine and subterranean conductors, however, the conditions are such, for reasons we need not at present enter upon, that the variable period is generally very much greater than that for overhead wires of the same length. The Thomson recorder shows this very clearly, for supposing that the current received on a long cable arrived immediately at its full strength, and that the coil and siphon responded immediately—that is to say, had no retarding friction or inertia—then the received signals would always be as shown in Fig. 16, and the application of a permanent current would produce a right-angled outline like *a b c*; but owing to the gradual rise and fall of the

current during the variable periods preceding and succeeding an application of the battery to the sending end, the slurred signals and curved line, *d e f* (Fig. 16A), are actually received. On a long submarine cable the current which passes from the cable to earth one second after the application of the battery to the distant end may be perhaps only 40 per cent. of the full ultimate current. After three seconds the current will probably be very nearly at its full strength, but many seconds must elapse before the maximum can be said to have been reached.

46. **Velocity of Electricity Variable.**—It is clear, therefore, that it is incorrect to speak of the velocity of electricity. If, for instance, an indicating instrument be set at the receiving end of a line, and the time which



FIGS. 16 AND 16A.*

elapses between the application of a battery at the sending end and the indication of this instrument be measured, we can say that this signal required that particular interval of time to be produced over that distance of line under those circumstances; but by altering the sensitiveness of the instrument and the form, dimensions, and position of the line we can make the time required to transmit that signal as great or as small as we please; and, so far as is known, there is no time of transmission over any given distance so short that by still further altering the conditions it might not be made yet shorter; so that, under these circumstances, it is impossible to assign any velocity as inherent to electricity, like that which we attribute, for example, to light in its passage through space.

* The curve *d e f* is accurately given on a larger scale at p. 329, Jenkin's "Electricity and Magnetism," 2nd Edition.

47. Period of Constant Flow of Current. — After the variable period has passed, however, the current in a circuit will continue to flow, under the control of much more simple laws, at a constant rate, so long as the conditions of that circuit remain constant.

48. The Strength of Current is Proportional to the Conductivity of the Circuit when the E.M.F. in that Circuit is Constant.—This almost follows from our conception of the term conductivity, for if a certain circuit offers a given degree of facility to the passage of a current supplied by a constant E.M.F., then it follows that by doubling that degree of facility we double the current flow, and if we make the conductivity 0·2, 0·9, 15, or 1,000 times what it was at first, we shall with the same E.M.F. obtain 0·2, 0·9, 15, or 1,000 times our first current.

49. Current Strength Proportional to E.M.F. when Conductivity Constant.—When the conductivity of a circuit is constant, then the strength of the current which flows through it is proportional to the E.M.F., just as the flow of water through a pipe is proportional to the difference of level or "head," which drives it. So that by doubling the E.M.F. in a circuit of constant conductivity we double the amount of current flowing through the circuit, and if we make the E.M.F. 0·005, 0·7, 16, or 1,000 times what it was at first with the same conductivity, we shall get 0·005, 0·7, 16, or 1,000 times the original strength of current.

50. Practical Unit of Current: The Ampere.—The unit strength of current is called the *ampere*, after the French physicist of that name. It is the current which flows in the circuit of unit conductivity containing the unit E.M.F.—that is to say, one volt in a circuit of one mho generates a current of one ampere. It follows, therefore, from what we have seen in para. 48, that the current in a circuit of one volt and two mhos would be two amperes, with one volt and 50 mhos, 50 amperes; also by what follows in para. 49 that with two volts and 50 mhos it would be 100 amperes, with 50 volts and 50 mhos 2,500 amperes, and in the same

way with $\frac{1}{2}$ a volt and $\frac{1}{4}$ of a mho $\frac{1}{8}$th of an ampere; so that in all cases the current in a circuit is found in amperes by multiplying the E.M.F. in volts by the conductivity in mhos. This simple and vitally important statement of facts is named, after its discoverer, Ohm's law. Since, however, we have seen in para. 30 that the conductivity of a circuit is the same in mhos as unity divided by the number of ohms in that circuit, we may, to find the current, multiply the E.M.F., not by the number of mhos, but by their equivalent unity divided by the ohms in the circuit's resistance; in other words, divide the E.M.F. by the resistance. Thus we have seen that a circuit containing five volts and of 0·025 mho conductivity would sustain a current of $5 \times 0{\cdot}025$, or 0·125 amperes. But the resistance of the circuit would be $\frac{1}{0{\cdot}025}$, that is, 40 ohms, so that we arrive at the same result if, instead of multiplying 5 by the conductivity 0·025, we divide it by the resistance 40, that is $\frac{5}{40}$ or $\frac{1}{8}$, or 0·125. So that, as it is much more usual to speak about a circuit's resistance in ohms than its conductivity in mhos, the common expression of Ohm's law is that the current in a circuit is obtained in amperes by dividing the E.M.F. by the resistance. Thus, in a circuit of 100 volts and 1,000 ohms, the current would be $\frac{100}{1000}$, or 0·1 ampere, and by our previous rule, the conductivity being $\frac{1}{1000}$, that is, 0·001 mho, the current would be $100 \times 0{\cdot}001$, or 0·1; of course, precisely the same result.

51. **Unit Quantity of Electricity: The Coulomb.**—This unit current or ampere is that which transmits the unit quantity of electricity in one second. The unit of electric quantity is called the coulomb, and just as the unit flow of water through a pipe might be taken as that which allowed one gallon of water to pass any point in the pipe during one second of time, so the ampere is the strength of current, the rapidity of flow, which allows one coulomb to pass any point in the circuit during one second. So that if a constant current

of one ampere has been flowing for 100 seconds in a circuit, then we know that 100 coulombs of electricity have passed any point in the circuit during that time. In the same way a steady current of five amperes flowing for one minute causes a total transmission of 5×60, or 300 coulombs, and generally the quantity of electricity in coulombs transmitted by a constant current is the product of that current and the number of seconds it has been in flow.

52. **The Milliampere.**—For convenience in telegraphy a submultiple of the unit, the milliampere, is in very common use; it is the thousandth part of an ampere. To give an idea of what a milliampere practically is in telegraphy, it may be mentioned that on Morse land line circuits receiving becomes difficult to adjust and sustain when the received working current falls below one milliampere, which may be regarded as a minimum working current, while 5 milliamperes—that is, 0·005 amperes—is a good average working current. The current which enters the line at the sending end may be greater, and in the case of a long line in wet weather perhaps much greater, than this; but the difference escapes by leakage at the insulators along the line, just as a quantity of water forced into a long pipe containing many small leaks would be much reduced in amount before its issue at the distant end.

53. **Calculation of Strength of Current by Ohm's Law.**—The working current in a telegraph circuit whose insulation is good—so that the current escaping by leakage need not be considered—is easily found by Ohm's law. Suppose, for example, that we have an overhead land line 50 miles long, of 13ω resistance per mile, worked in the manner represented at Fig. 17—that is, on the open circuit principle, where no current passes through the line, except in actual signalling. The stations A and B are each provided with a Morse key, K, for sending the currents, a galvanoscope, G, of 30ω resistance, for observing that those currents duly pass, a Morse inkwriter or sounder, M, of 200ω resistance, and a battery, E, of 11 volts and 200ω total internal resistance. The levers of the keys are normally kept back by the spring p on the receiving contacts r.

Under these circumstances there is a complete circuit from A over the line returning by the earth and both receiving instruments, but in this circuit there is no battery. When A. depresses his key, however, the circuit is completed no longer through the back stop and receiving instrument but through the front stop *s* and battery E, and a current from the latter therefore flows to line and actuates the galvanoscopes and instrument S. While A's key remains depressed we know from Ohm's law that the strength of current at any point in the circuit—there being assumed no leakage—will be the total

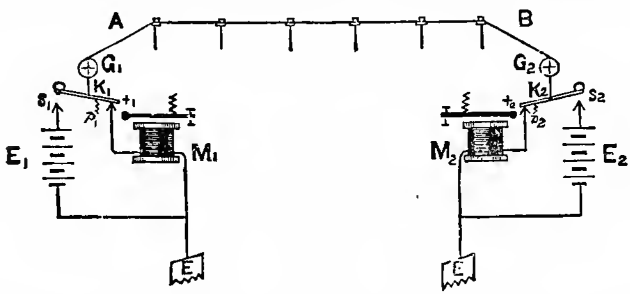


FIG. 17.

E.M.F., namely 11, divided by the total resistance, which we can sum thus—

A.'s battery B	200ω
A.'s galvanoscope G	30ω
The line A B—	
Fifty miles at 13ω per mile	650ω
B.'s galvanoscope G	30ω
B.'s instrument M	200ω
The earth connections and the earth perhaps	1ω
Total	1,111ω

The current flowing will therefore be

$$\frac{11}{1111},$$

or $\frac{1}{101}$th part of one ampere—that is, 0·0091 ampere or 9·1 milliamperes.

CHAPTER VI.

CURRENT INDICATORS.

54. **General Uses.**—The laws of the current, and the relation between E.M.F., resistance, and current having been considered, we shall now examine some of the instruments used to indicate the presence of a current, these being termed *detectors* or *indicators*, in distinction from galvanometers or current measurers. It will also be shown how detectors may be calibrated, so that their indications may be made to show the exact strengths of current passing through them.

Detectors are in constant use on land-line telegraph circuits, one being fitted to each Morse instrument, so that when the operator is sending a message he may observe that his current is going out to line all right. And when the distant station calls up, the call is shown on the detector as well as the Morse armature, so that if the latter should fail for any reason there will still be always a visual indication attracting the attention of the operator.

Detectors are very useful for tracing a fault on an instrument or line, or for identifying certain wires from a large number. The manner of doing these will be explained as soon as the instrument itself has been considered.

55. **Direction of Current.**—One of the most useful and common forms is shown in Fig. 18, which will indicate the *direction* of a current by the needle being deflected one side or the other. It is advisable to ascertain which way the needle moves with a given direction of current through it,

as we shall require to know this frequently when testing with the instrument. Suppose that on connecting it to a voltaic cell, as in Fig. 19, the needlo moved to the right, then, knowing that the current leaves the carbon plate in the cell and enters the detector by its right-hand terminal, as shown by the arrow, we should know that the needle would always deflect towards the terminal where the current entered, or towards the positive terminal. For if we reversed the wires on the detector, so that the current entered by the left-hand terminal, the needle also would be deflected to the left.

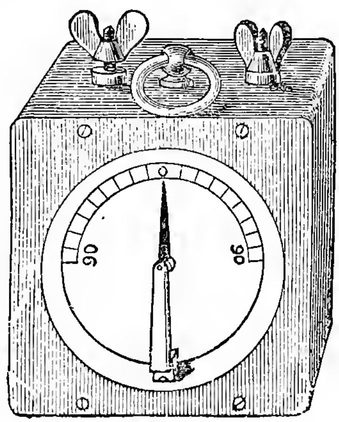


Fig. 18.

56. **Direction of Deflection.**—All instruments do not deflect the same way for the same direction of current through them. The direction in which the needle of an instrument will deflect when a given current is supplied to it depends on three things:—

1. The position of the poles of the needle relatively to the coil when the former is at rest.
2. The direction of winding of the instrument coils.
3. The connecting up of these coils to the instrument terminals.

As an illustration, we might cause the deflection of the above detector in Fig. 19 to be always towards the *negative* terminal by altering *either* of the three conditions mentioned. But if we altered the instrument by any two of these conditions, say the position of the needle at rest, and the connecting up of the coils to the terminals, we should not alter the direction of deflection for a given current, since these two changes would neutralise each other. Similarly, if we altered all three conditions, we should *reverse* the deflection.

57. **Mounting of Needles in Vertical Detector.**—There are two needles to the instrument—one, the smaller, N S, being of hard steel well magnetised and weighted at the end N, this end

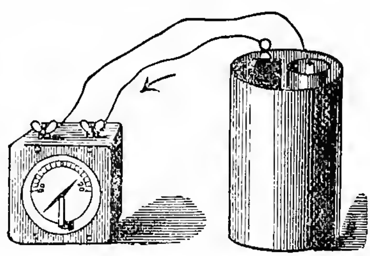


FIG. 19.

being made broader. The use of this is to bring the needles back to zero after being deflected, or, in other words, to act as a controlling force (viz., that of gravity on the larger end) to the system. Fig. 20 shows the needles and spindle detached from the instrument. The longer needle, $n\,s$, which is seen outside in Fig. 18, has its poles reversed to those of the inner needle N S. The reason of this is that when a current is passed through the coil a magnetic field of force is produced whose direction of flow is through the interior of the coil and back again round the outside in a complete circuit. Hence the direction of the force on a magnetised needle placed *inside* the coil is opposite to that exerted upon it when placed outside; and the needles, which are fixed parallel to each other, must therefore have their similar poles pointing opposite ways,

in order that both needles may be deflected in the *same direction* by magnetic forces in opposite directions. This will be explained more in detail when galvanometers are considered.

Both needles should be perfectly parallel, to ensure which each is pierced with a square hole at its centre, which fits over a square key on the brass hubs B B; and the latter, having a very fine thread cut, permits of tightening up the needles securely in position by a little square-headed nut (shown at A A), which can be screwed on by means of a small pair of pliers.

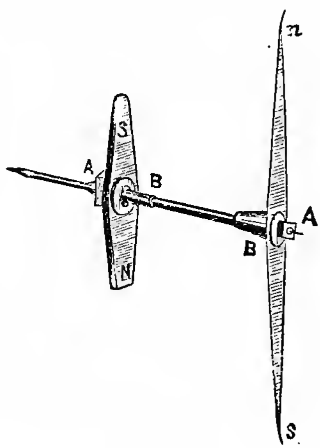


Fig. 20.

58. **Mounting of Coils.**—Turning now to the interior of the instrument, the two coils are wound on brass bobbins, one of which is shown mounted in position in Fig. 21, the other having been removed to expose the needle, N S, to view. The mounting of the spindle can also be seen, and the freedom of the latter to turn can be adjusted to great delicacy by the screw, T. The interior space of the coils in which the needle moves is very little larger than the needle itself; which brings as much turning force as possible to bear on the needle when the current is on; this makes it important to fix the coils

accurately in position, so that the needle may not touch the sides during any part of its play. The brass bobbins are therefore screwed on to the brass plate P, with little brass pins to act as guides and keep them in position. The screw hole of the removed coil is seen at H and the three guide holes round it. The brass angle pieces B B serve to screw the plate and coils when adjusted inside the wooden box of the instrument.

59. **Direction of Winding of Coils.**—Regarding the winding of the coils, it should be remembered that the current must

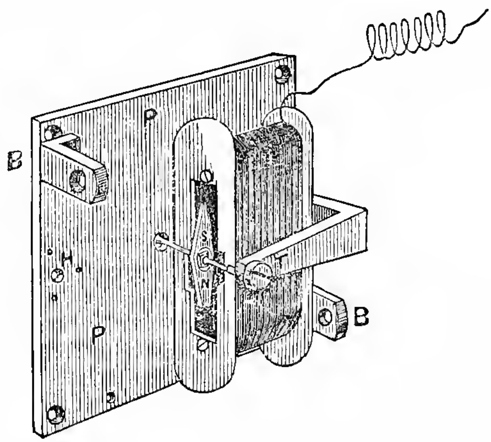


Fig. 21.

flow through both coils in the same direction, since the two coils should form virtually one electro-magnet; in fact, the reason for the electro-magnet being made in two coils is only for facility of construction and for getting at the needle conveniently. In order to be able to detach each coil separately from the instrument (for convenience of adjustment) the coils are not connected by a wire between them, but the underneath end of each coil is soldered on to the brass bobbin. So that, before starting to wind a coil, the end of the wire is soldered to the brass bobbin, and when the coils are finished and

screwed into their places electrical contact is made between the two *inside* ends of the coils through the brass work.

60. **Coils Connected "in Series."**—Now, if we intend the coils to be joined electrically in series, a little thought will show that they should be wound in opposite ways. Referring to Fig. 22, each coil when its winding was commenced had the underneath end soldered at C C^1 to the bobbin, the winding on each coil then being in opposite ways. If, now, a current is sent, say from terminal A to B, it will flow round the two coils in series, and in the

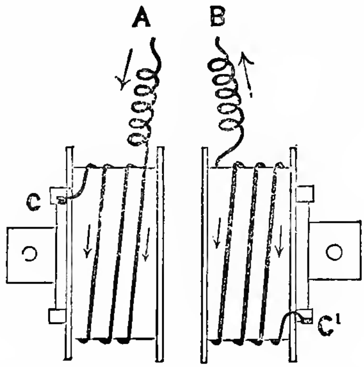


Fig. 22.

same direction through each, the points C C^1 being connected through the brass work.

61. **Coils Connected in "Multiple Arc."**—If we required to reduce the resistance of the coils, still utilising both, we should connect them in parallel or multiple arc, their combined resistance being then reduced to one quarter (para. 41). To do this we could not alter the connections of the coils in Fig. 22 without having wires between them to connect them electrically. The inside end of either coil, say that end attached to C_1, would have to be removed and connected to the outside end of the other coil, a short wire, *w*, being used if the inside end exposed was too short (*see* Fig. 23).

Then the outside end of the first named coil would have a wire connecting it to the brass work at any convenient point, say at C_2. Now, passing a current from A to B, it is seen to flow through both coils in "parallel," and in the same direction through each, and meet at the brass work C C_2, whence it passes to B.

If, when the coils were being wound, they were intended to be used in parallel, we should solder both inner ends to the brass frame, as before; but the coils would be wound the *same*

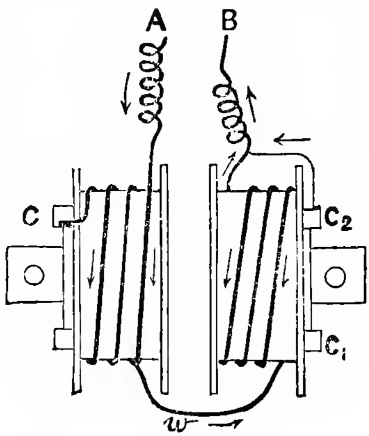


FIG. 23.

way, their outer ends, joined together, forming one terminal, A, and the other terminal, B, being a connection simply to the brass framework (*see* Fig. 24). It will then be seen that the current traverses both sides simultaneously and in the same direction.

62. Relation between the Sensitiveness of a Detector and its Resistance.—The resistance of the coils is an important point as regards the sensitiveness of the instrument when we come to employ it with batteries of different resistance. For instance, any number of similar voltaic cells connected in series would give the same deflection on the detector as a single one of

those cells if their internal resistance was high and the resistance of the instrument coils very low. No idea could be formed, therefore, from such a test as to the number of cells connected together.

To take an example—a Minotto cell (*see* para. 17) measures when in actual work about 1 volt E.M.F., and 20ω internal resistance. Now, if we connect this to a low-resistance detector—say $\frac{1}{10}\omega$—the current through the instrument will be $\frac{1}{20{\cdot}1}$ ampere, or 49·7 milliamperes; and if we connect ten of

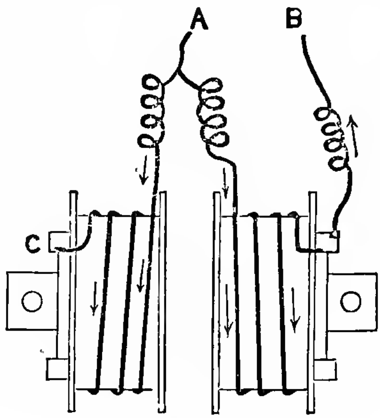


Fig. 24.

those cells in series the current will be $\frac{10}{200{\cdot}1}$ ampere, or 49·9 milliamperes. Now, as the current is practically the same in the two cases, the deflections on the instrument will be equal, and therefore we should be unable to tell from the indication of the instrument alone what battery power was on.

If, on the other hand, the resistance of the coils was higher, say 100ω, we should get with one cell $\frac{1}{20+100}$ ampere, or 8·3 milliamperes, and with ten cells in series $\frac{10}{200+100}$ ampere, or

33 milliamperes—*i.e.*, about four times as much current, and therefore a sufficient diffeɪence in the deflections.

Similarly, if low-resistance batteries, connected in multiple arc, were tested with a high-resistance detector, the deflection would be the same for any number of cells. As an example, take large-size bichromate cells which will measure about 2 volts E.M.F. and 1ω resistance per cell, and let the detector coils measure 100ω. One cell connected to the instrument would give $\frac{2}{1+100}$ ampere, or 19·8 milliamperes; and ten cells in multiple arc would give $\frac{2}{0{\cdot}1+100}$ ampere, or 19·9 milliamperes, and therefore there would be no difference in the deflection.

On the other hand, if we employ a low-resistance detector, say one of $\frac{1}{10}\omega$, one cell would give $\frac{2}{1+0{\cdot}1} = 1{\cdot}82$ ampere, and ten cells in multiple arc $\frac{2}{0{\cdot}1+0{\cdot}1} = \frac{2}{0{\cdot}2} = 10$ amperes, giving considerable difference in the currents, and therefore in the deflections.

Speaking generally, therefore, we should conclude that the more nearly the resistance of the detector coils approximated to the internal resistance of the batteries tested, the more sensitive the instrument would be. We must, however, go further than this, and see what are the exact relations of the resistances for maximum sensitiveness.

63. Exact Determination of Best Resistance of Coil.—When we pass a current through a coil of wire we set up inside, and in the space immediately surrounding that coil, what is termed a "field of force." The physical properties and delineation of this field will be considered more in detail subsequently, but for the present it will suffice to say that this field resembles that surrounding a permanent magnet, and exerts a turning force on any pivoted magnetic needle in the centre of the coil, tending to draw it into a line parallel to the axis of the coil. The actual *force* exerted on the needle depends

as much on the strength of the needle poles as on the strength of the field created at the point where the needle is placed—in fact, is the product of these two factors. The stronger the field of force created in the coil the larger the deflection of the needle will be. We have therefore to consider what should be the conditions of winding and resistance of the coils to give the strongest field when any source of E.M.F., such as a battery of known resistance, is employed to send a current through it.

Now the strength of field produced at the centre of a coil varies as the strength of current, also as the total length of wire wound on, and inversely as the square of the mean distance of the wire from the needle, or, if the coil is circular and the needle suspended at its centre, inversely as the square of the mean radius.

Suppose now that the size of the bobbins and the space to be occupied by the wire when wound on has been fixed upon; then the strength of magnetic field of the coils will vary simply as the product of the current through them and the square root of their resistance. The latter will be clear on considering that for a given weight or volume of wire the resistance is proportional to the square of the length. As an example of this, if we have two lengths, A and B, of different sized wire, B being ten times as long as A, but weighing exactly the same, it is clear that the sectional area of B must be $\frac{1}{10}$ that of A. Since B is ten times the length, its resistance would also be ten times that of A if it was of the same size; but since it is only $\frac{1}{10}$ the area of A, its resistance is increased tenfold more, and therefore B measures 100 times the resistance of A—in other words, the resistance varies as the square of the length when the weight or volume is constant, or conversely the length varies as the square root of the resistance for a given weight.

We have then, as above, the field of the coil proportional to

current × length of wire, for which we may write

$$\frac{\sqrt{\text{resistance of coil}}}{\text{sum of resistance of circuit}},$$

the current being inversely proportional to the total resistance of the circuit (viz., battery, leading-wires, and detector coil), with a constant E.M.F. This expression for the field can be easily shown mathematically to become a maximum when the resistance of the coil is equal to the battery and leading-wires, but it can be shown equally as well graphically without the aid of mathematics by the plotting of a curve from a few worked-out values. For instance, let us see how the field varies for different resistances of the detector coil when the total resistance *outside* the detector is 100ω.

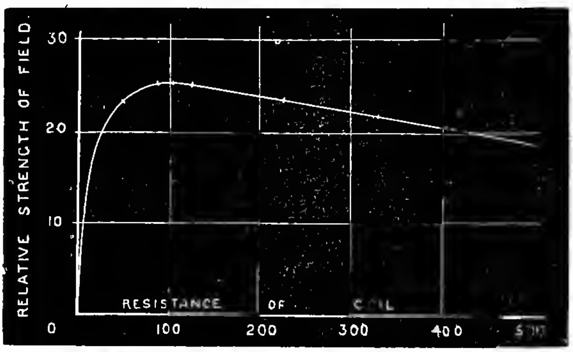


Fig. 25.

Taking tho resistance of the coil as 9ω the field will be

$$\frac{\sqrt{9}}{100+9} = \cdot 0275,$$

and taking it successively as 49, 81, 100, 121, 225, 324, 400, and 484 (these numbers having simple square roots), we find the field becomes respectively ·047, ·0497, ·050, ·0497, ·046, ·0424, ·040, ·0376. These numbers are not in any particular units; they simply give the *relative* strengths of field. Multiplying each by some large number, say 500, will not alter their relation to each other, and will give us whole numbers instead of fractions, which is more convenient for plotting the curve. They will then become respectively 13·7, 23·5, 24·8, 25, 24·8, 23, 21·2, 20, and 18·8.

Marking off these values on the vertical ordinate (*see* Fig. 25) and the corresponding resistances given to the detector coil

on the horizontal ordinate, we get the curve shown, which indicates that the maximum strength of field, and therefore the maximum deflection, is produced when the resistance of the coil is 100ω, *i.e.*, the same as the resistance of the rest of the circuit.

The curve shows also that if the actual resistance is any given fraction of the best resistance, say one-fourth, the field is the same as with the same multiple of it, viz., four times, both of which in this case give a field of 20. Or a 10ω coil detector would give the same deflection as one wound to $1,000\omega$ for the same size of coil, but the field would only be (by reference to the curve) $\frac{15}{25}$, or three-fifths of the maximum.

The above figures are all supposing 100ω to be the total resistance in circuit *external* to, or *outside*, the instrument, but the curve will necessarily be of the same shape for any resistance taken, and will always show the field, and therefore the deflection, to be greatest when the coil (of given size and given weight of wire) is equal in resistance to the rest of the circuit.

64. Detectors for Telegraph Work.—For telegraph work, where comparatively high resistance batteries and instruments are met with, the resistance in a detector circuit when used for a battery or line test is more frequently over 100ω than under; they are therefore mostly used from about 200 to 500 ohms. But occasionally much lower resistance batteries require testing—such, for instance, as Thomson's tray cells or Fuller's bichromate cells—when a lower resistance coil, say of 5ω or 10ω, will be more sensitive. Some detectors are wound with two coils of widely different resistances for this purpose.

A portable detector, the size of an ordinary watch, has recently been brought out by Messrs. P. Jolin and Co., of Bristol, which is wound with two coils, one of fine wire measuring $200{\cdot}6\omega$, and the other of thick wire being $0{\cdot}65\omega$. There are two bobbins, which are first wound with the fine wire coil, the underneath end of the wire being soldered to the

frame, and the upper end attached to one terminal, marked Z (*see* Fig. 26). To use this high resistance coil alone, therefore, the battery wires must be connected, one to the outside case of the detector and the other to terminal Z, the winding being so arranged that the needle points towards Z when this terminal is connected to the zinc pole. The thick wire coil is wound on next, one end being soldered to the brass bobbins as before, and the free end taken to the other terminal. If, then, we connect the battery wires between this terminal and the case the thick wire coil alone is in circuit. It would really only be the

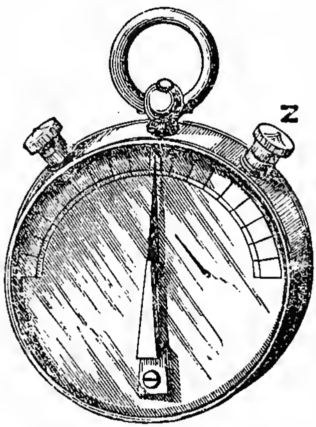


Fig. 26.

low-resistance coil that we should require to use alone; when the high-resistance coil was required we should use both coils in series by connecting the battery to the two terminals, by which we should get the advantage of the extra turns of wire and an inappreciable increase in resistance.

65. Detectors for Telephone Work.—As regards telephone circuits and instruments, the detector is an indispensable item of an inspector's kit, as it is frequently required to detect any fault in his instruments or to examine the condition of the batteries. The more portable,

while reliable, of course the better, and some of the telephone companies have in use the pocket type described. As the batteries used are generally Leclanché's and the resistances met with rather under than over 50ω, the detector would be wound to that resistance or less, according to the resistance of the induction coils and receivers and the type of battery in use.

66. **Indicators for Large Currents.**—For dynamo or accumulator circuits where the resistances are very small, detectors for showing the direction of the current, or ascertaining whether it is constant, are in use, and are wound with a few turns of thick wire to resistances from 1ω to $\frac{1}{100}$ω, or even less. In fact, with ordinary electric lighting currents, say from 10 amperes upwards, no coil at all is required for finding the *direction* of the current; an ordinary pocket compass placed over the main wire at any part of the circuit will show the direction of the current at once. The compass-needle will try to set itself at right angles to the wire in which the current is passing. If the north pole of the needle turns to the *right* the current will be flowing *away* from the observer, and *vice versâ* (*see* Fig. 27). It will easily be seen whether the current is strong by the rapidity of the oscillations made by the needle when placed over the main wire; and if the current is a feeble one, causing very sluggish oscillations, as it would under the influence of the earth's field alone, a portion of the main wire should be selected which lies north and south, or a few inches of it may be easily bent into this direction before applying the compass test; it will then be seen quite clearly towards which direction the needle tends to turn from the meridian, the earth's field tending to keep it parallel to the wire, and the field produced by the current tending to move it away from a parallel position.

On this principle an engine-room current indicator is easily constructed with a magnetic needle freely suspended over a single broad band of copper, the ends of which are attached to terminals outside the instrument. The writer

recollects seeing an indicator of this kind, with a vertically-suspended needle placed where it could be readily seen, in the engine room at one of the London exhibitions. It was in the circuit of six arc lamps, which served to illuminate the grounds from the top of a high mast, and the dynamo attendant could see by it quite well whether his lamps were burning all right or if one was failing. There being no bobbins or coils, the instrument is extremely simple to make, and, if required, can

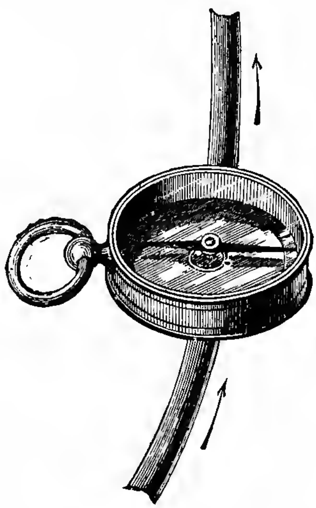


FIG. 27.

be calibrated in amperes by comparison with some ammeter whose constant is known. One is illustrated in Fig. 28, the copper or brass band behind the needle being shaded, and the needle itself having its north end downwards. The direction of deflection for a given current is then as shown.

67. **Effect of Current in Leading Wires.**—A question that will naturally suggest itself from the last few observations will be: Does not the current in the leading wires attached to a *measuring* instrument affect its readings? As a matter of fact it does so with a weak controlling field, when a large current is

being measured, and the precaution must be taken of twisting the two leading wires as they approach the instrument, so neutralising the extraneous field they produce. If, however, the instrument has a powerful magnetic controlling field the effect is practically *nil*, and there is no necessity for the above precaution. In all cases, before readings are taken, it can be noticed whether, when the current is on, the leading wires affect the needle by shifting their position relatively to the latter, and observing whether any alteration occurs in the number of degrees deflection.

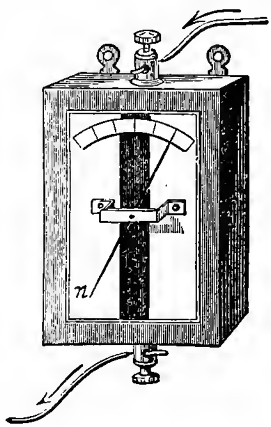


FIG. 28.

68. **Indicators with Coils of Different Resistances.**—Large telegraph stations and telephone exchanges have in use a considerable number of cells, in some cases of different types, for local circuits or line work, and the battery man has to keep a spare set of cells ready to put on to a circuit in case any should "run down." In cable stations there are in addition sets of very low-resistance "tray" cells for magnetising the electro-magnets of the recorder instrument. An indicator with coils of different resistance for testing the condition of cells of different types is then a requisite. A form of indicator fitted in this way (by Messrs. Elliott Brothers)

is shown in Fig. 29, the coils being respectively of 2, 10, and 1,000 ohms. Batteries or circuits to be tested are connected to the instrument at A and B, and the plug P is inserted in the hole opposite the number showing the

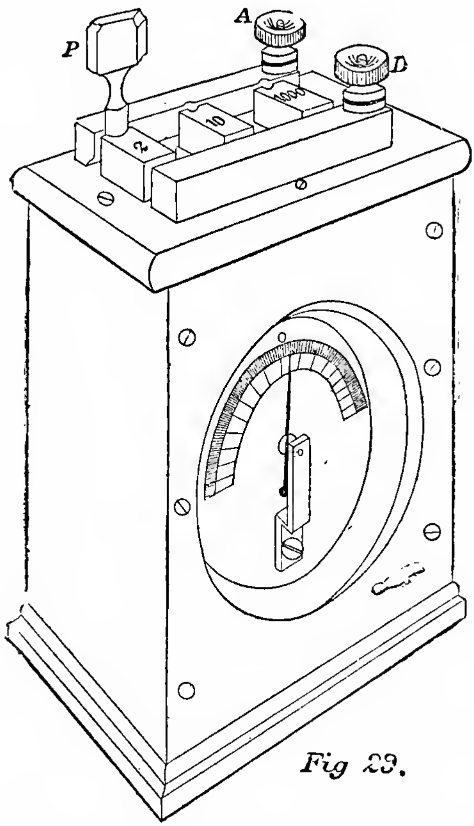


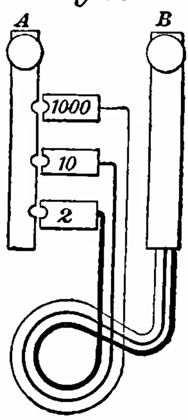
Fig 29.

resistance of the coil required. This arrangement of brass bars and movable plug is technically called a "*commutator*;" a plan of this portion is shown in Fig. 30 with the connections to the coils. The current can be made to pass from one

terminal to the other through any of the coils by inserting the plug at the bar marked with its resistance. The high-resistance coil is wound with many turns of fine wire, and the two-ohm coil with a few turns of thick wire.

69. **Effect of Loss of Magnetism in Needle of Vertical Indicator.**—Should the needle of a vertical detector lose its strength of magnetism to any extent, the deflection given by the instrument with a given current will be weakened accordingly, but as the north pole is the weighted or lower end the needle lies in a favourable position as regards the earth's field

Fig 30.



in this latitude for maintaining its polarity, and it will probably be years before its magnetism becomes sensibly weakened. If strong currents, however, are put through the instrument, especially if the circuit is disconnected and connected several times in succession, the magnetism of the needle may become affected. This is the more noticeable in telegraph needle signalling instruments, which are connected to a long length of overhead wire. Atmospheric electrical discharges take place at times through the wire, and if the instrument protectors do not act, the needle may become partially demag-

netised, or even have its magnetism reversed. Devices for preventing this will be considered later on in this series when telegraph apparatus is examined. The needle signalling instrument, however, is mentioned at this point because its internal construction is exactly the same as the vertical indicators which have just been considered.

70. Effect of Loss of Magnetism in Needle of Horizontal Indicator.—When the needle of an indicator is pivoted so as to lie horizontally, any weakening of its magnetism has very little effect on the sensibility of the instrument. The reason for this is that the controlling force, or force tending to maintain the needle at zero, is that of the earth's field, the actual amount of that force acting at each end of the needle being the product of the earth's field and the strength of one of the needle poles. On the other hand, the force exerted at each end of the needle by the current in the coils, tending to move it away from zero, is equal to the product of the field of the coil and the strength of one pole of the needle. Since both the moving force and the controlling force depend equally on the magnetic strength of the needle, the ratio between them is simply the ratio of the respective fields of the coil and the earth, and the needle will therefore take up a position depending on the relation between the two fields irrespective of the actual strength of the needle poles; unless, indeed, they are entirely demagnetised, in which case there would be no moving or controlling force at all, except, probably, irregular forces caused by magnetic induction.

71. Effect of Friction on Pivot.—The friction on the pivot, however, which is a negligible quantity when the needle is strongly magnetised, becomes an appreciable obstacle to its movement when its magnetism is weakened, for while the *relation* between the moving and controlling forces depends simply on the two fields, and therefore is unaltered by weakening the magnet, it will be seen by what was said above that the *actual amount* of these forces is proportionately reduced as the magnet becomes weaker, and that

therefore the needle is not drawn towards or held in its deflected position with the same force and with the same

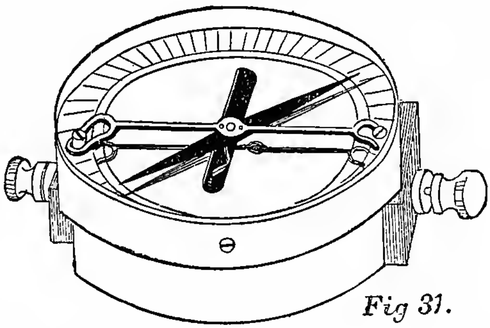


Fig 31.

facility of overcoming friction as when it is strongly magnetised.

72. **Horizontal Indicators.** — The horizontal pivoting of needles, however, admits of very sensitive mounting of the

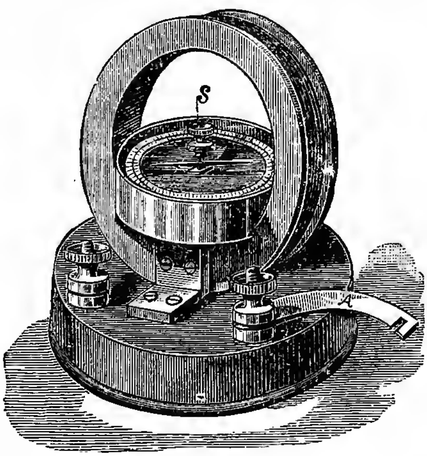


FIG. 32.

needle spindle. Illustrated in Figs. 31 and 32 are two very sensitive indicators of this class by Messrs. Elliott Bros.

In Fig. 31 the spindle joining the two parallel magnetic needles, whose opposite poles point the same way, is pivoted in jewel centres, and constitutes an indicator that may be very well used, with proper calibration, for a number of useful tests requiring accuracy. The dial is about $3\frac{1}{2}$in. diameter, and the coil is wound to 1,000ω resistance.

There is only one magnetic needle in the instrument represented in Fig. 32, the pointer, which indicates the deflection, and is fixed at right angles to the needle, being non-magnetic. When the instrument is used the coils must be turned round until they lie in the same plane as the needle *n s* (*see* Fig. 33).

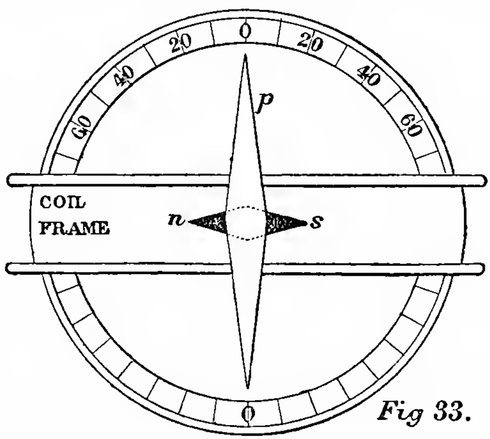


Fig 33.

Now, as it would be impossible to read small deflections with the coil impeding the view, a non-magnetic pointer, *p*, is attached at right angles to the needle and points to zero when the instrument is ready for use, its angular deflections being of course the same as those of the needle. The screw seen at *s* in Fig. 32 is fixed to the glass lid of the needle box, serving to remove it when required. The metal arm A serves to connect together the two terminals, or, as it is termed, "*short-circuit*" the instrument. This might be required if the instrument was connected to a circuit where a current was con-

tinually flowing, the short-circuit being removed every time a reading had to be taken, and then replaced, so that the current would only pass through the coils when a reading was required. The coil of this indicator is about 4in. outside diameter, and is wound to a low resistance. When an indicator is set so that the plane of its coil is parallel to the needle *at rest*, a deflection of 90deg. can never be reached, as this would require an *infinite* current, and, moreover, the instrument is so insensitive to slight changes of current when the deflection is above 75deg. that deflections are usually kept below this, if possible, when a test is taken involving reading two deflections representing two strengths of current nearly the same; in fact, the readings are most sensitive when the needle is at an angle of 45deg. For testing, where slight changes in current have to be read, the deflection should therefore be first brought to this angle, either by altering the resistance in the circuit or by shunting the instrument.

CHAPTER VII.

SIMPLE TESTS WITH INDICATORS.

73. **Tests for "Continuity."**—Before leaving the subject of current indicators, or detectors, and proceeding to the consideration of current measurers, or galvanometers, we will proceed to describe the methods of using the former instruments for some ordinary simple tests.

The most frequent use to which a detector is put is to ascertain whether a given circuit is metallically continuous, or, in other words, to test the circuit "for continuity."

In all electrical circuits containing apparatus in which certain contacts are "made" and "broken" at intervals, or continuous electrical contacts are made through moving parts of apparatus, such as the lever and axle of a switch or key or the carbon rod of an arc lamp, there exists a liability to loss of continuity through these contacts, either owing to their not being sufficiently clean, wearing away, oxidation, insufficient pressure between the surfaces of contact, or gradual fusing of the contact points as the result of sparking. Platinum is used for contacts, as it is a hard metal, and does not oxidise. As regards pressure between contacts a wheel or rolling contact is not so reliable as a rubbing contact where it is required to make continuous connection with a moving part. Switches for altering battery power or changing the connections of circuits are not considered reliable if they make more than three contacts at the same time. They are sometimes provided with screws for adjusting the pressure

after the switch lever is in position, or, as in Hedges' form of switches for large currents, small independent springs exert a continual pressure between the lever and the contact. A later device to ensure never-failing contact in a lighting circuit is to make the lever actually cut into the contact brass blocks, which latter are made of such a form that the wear can easily be taken up.

Where the pressure exerted is manual, as in the case of signalling keys in telegraphy, the operator easily acquires the difference in pressure requisite for working a long or short cable.

If sparking occurs between two contacts, such as would be the case if the contacts were required to effect intermittent short-circuit across the terminals of an electro-magnet through which a current was passing, or "make" and "break" its circuit, a high resistance, say of carbon connected permanently between the contacts, offers a path for the "extra current" from the electro-magnet, reducing the sparking and preserving the contact surfaces. It is seldom that wires connecting together parts of apparatus or leading wires running between different rooms of a telegraph or telephone office actually break, though they may do so in some cases, as for instance, if exposed to the corrosive effect of acid at any point, or if led round a sharp corner and subjected to strain or abrasion. But it is not at all an unfrequent occurrence for a contact to fail between a wire and the terminal to which it is screwed. Either the terminal is too large for the wire and does not properly grip it, or the wire has been bent in a loop round the terminal the wrong way—that is, in the direction *opposite* to that in which the screw turns, so that any extra tightening of the screw may force the wire out altogether; or the wire may be attached to a battery terminal which has become corroded and surrounded with crystals, owing to the "creeping" of the battery liquid, in which case a bad contact would be made.

74. Detection of a Fault in Telegraph Apparatus.—It sometimes happens that through failure of some contact in his own *receiving* circuit connections, the operator at one end of

a telegraph line spends a long time in "calling up" the station at the other end, which, while answering all the "calls," is quite unable to speak to the station calling.

It is therefore one of the first considerations to look well after the receiving-circuit connections if no reply to a call is immediately forthcoming.

To illustrate this, a very simple example will now be taken of a fault of this kind occurring, say, in the receiving circuit of a single-current Morse signalling instrument. The

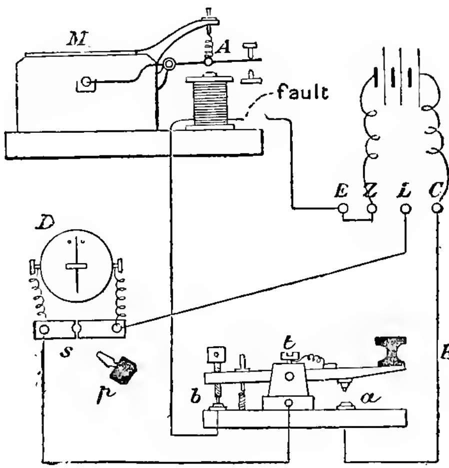


Fig. 34.

illustration (Fig. 34) represents the apparatus usually mounted and connected together, and consists of a Morse ink-writer, M, sending key K (for transmitting signals by hand), line detector D (which can be short-circuited when not required by plugging *p* in *s*), and four terminals marked earth, line, copper, and zinc. The sending battery is, of course, external to the instrument, and is connected to terminals C Z. The sending key is a brass lever, movable about the centre *t*. The lever is generally electrically connected to *t* by a short flexible spiral of wire, since a failure

of contact may occur at the axle about which the lever turns when manipulated. The back contact *b* is always closed when the key is at rest, the lever being kept in this position by the adjustable spring at the back. In this position it will be seen that, if there is no fault in the circuit, a current arriving from the line to the terminal L has a path open to it through the detector D, from *t* to *b* of the key, and through the electro-magnet of M, to the terminal E, and thence to earth, completing the circuit back through the earth to the sending battery at the distant station. If there is some fault or break in this circuit, the detector already attached to the instrument can be readily used for localising it.

We will suppose the sending circuit to be all right. If this is not known it can easily be tried by connecting a wire between E and L, which short-circuits the instrument, and then if D is deflected every time the key is pressed it proves this circuit and the battery to be all right. Now disconnect the wire between E and L, and test the receiving circuit by taking the battery wire off C and putting it on to L, which connects the battery direct to the receiving circuit. No deflection will be observed on D, as there is some fault in this circuit. Disconnect the other battery wire from Z, and hold it in the hand so as to be able to connect it to different points in the circuit. We know the circuit is right from L to the key at *t* from the previous test, so that touching the end of the wire held in the hand on the metal at *t* we should get a deflection on D. Now, suppose that a deflection is observed when the end of the wire is touched at the lower contact at *b*; this shows that the back contact of the key is good. Also let deflections be noticed on touching the wire to the beginning and end of the electro-magnet coil of M, showing that the connection from *b* to the coil and the coil itself are all right. There is only one place now where the fault can be, which is between the coil and the terminal E, and on closely examining this connection underneath the instrument the break would be found, probably caused by the drawer containing the paper slip having chafed against the spiral of thin wire from the coil.

What has taken some time to describe here is really only the work of a few seconds, and it is quite a simple operation with practice and a knowledge of the circuits to speedily localise in this manner any fault in any set of connections, however complicated.

For the many different continuity tests—such, for instance, as localising a fault on an instrument or a set of electrical apparatus of any kind, or for proving the continuity of the wiring of a building while the work is progressing, as is usually done in wiring a building for electric lighting to prove that everything is right before the lighting current is switched on—it is very convenient to have one of the forms of combined detector and battery made by many manufacturers. Both are contained in one small portable box, and connected up ready for use, with two terminals on the outside, which, when connected together, either directly or through some circuit being tested, cause the needle to be deflected.

75. **Tests for Identity of Wires.**—The detector is a ready means of identifying wires which are run together from, say, one part of a building to another, no distinguishing mark having been attached to them, or which have become mixed in passing together through walls or flooring. Gutta-percha covered wires for telegraph work are manufactured with one or more ridges of insulating material running along their entire length, in which case any particular wire can be identified at any point by feeling its surface. Wires insulated with this distinguishing mark are in use at some cable stations, and serve to connect the cables from the beach at low-water mark to the cable house or thence to the town office. They are drawn through iron pipes laid underground, and are kept filled with water to preserve the insulation. In this case any wire can easily be identified by the number of ridges on its exterior.

For electric bell work the outside cotton coverings of wires leading from the different rooms to the annunciator are sometimes differently coloured, so that anywhere throughout their length they can be identified.

Suppose, now, that it is required to identify six wires laid from a battery room to an office. We should start by numbering the wires at one end A (Fig. 35), and then connect, say, the copper pole of a cell to No. 1, and the zinc pole to No. 2. Now, on going to the other end, B, and trying the wires successively on the detector, the pair of wires that cause a deflection are, of course, Nos. 1 and 2; but this alone does not show which is which. This is determined by the *direction* of the deflection. Suppose that by a previous trial it had been ascertained that when a cell was connected up directly to the detector the needle was deflected *towards* the

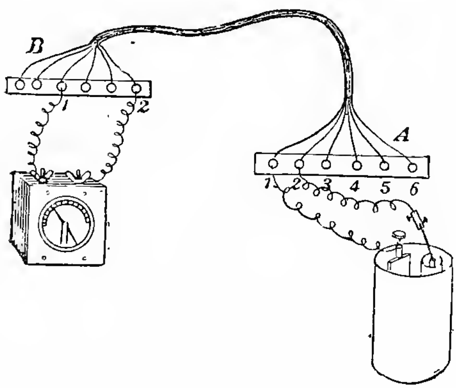


FIG. 35.

terminal at which the current *entered* the instrument (*see* Chap VI., para. 56) we should know that the needle would point towards the terminal connected to No. 1. We have therefore identified Nos. 1 and 2, and they should be so marked at once. No. 1 may now be kept attached to one pole of the cell in the battery room, and one terminal of the detector in the office, while all the other wires are identified in turn. Thus, No. 3, being connected to the other pole of the cell, the wire which when touched by the other terminal of the detector gives a deflection is No. 3, and so on, till the last but one is done, and then all are known.

76. Test for Insulation.—Overhead Line Insulators.—A sensitive indicator of the horizontal type described in para. 72 and illustrated in Fig. 31 may be used for testing the insulation of a circuit where it is not so important to know the exact resistance of the insulation when perfect as to keep up a daily inspection that it does not fall below a certain amount. An indicator of this type will give 10deg. deflection with one Leclanché cell and 150,000 ohms in the *external* circuit, the coil of the instrument itself measuring 1,000 ohms. With these high resistances, that of the cell would be practically negligible. Up to such a small angle as 10deg. the deflections may be

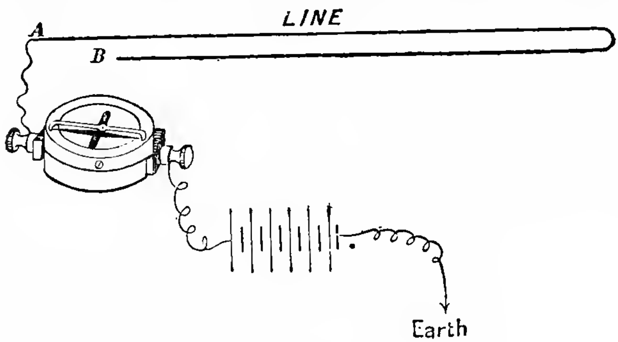


FIG. 36.

taken as proportional to the currents, and therefore inversely proportional to the total resistances of the circuit. That is, 1deg. deflection would represent (150,000 + 1,000) 10 = 1,510,000 ohms, or 1·51 megohms total resistance, which, deducting the resistance of the instrument, would be 1·509 megohms in the external circuit.

By increasing the number of cells the instrument could be made to read up to a much higher resistance.

The connections would be made as in Fig. 36, the battery being connected through the instrument to one end of the main wire whose insulation resistance is to be found, the other end, B, of the wire not touching anything, or as it is termed

left "free." The other pole of the battery must be connected to earth, this connection being easily made on to a water pipe which runs underground. If this connection is made permanently it must not be to a *lead* gas pipe, or indeed to any *lead* pipe; a plate of galvanised iron sunk in the ground is the best.

Now, let us suppose that the wire A B is an aerial one, and is run on poles either from roof to roof, as is the case in London with the electric lighting and telephone

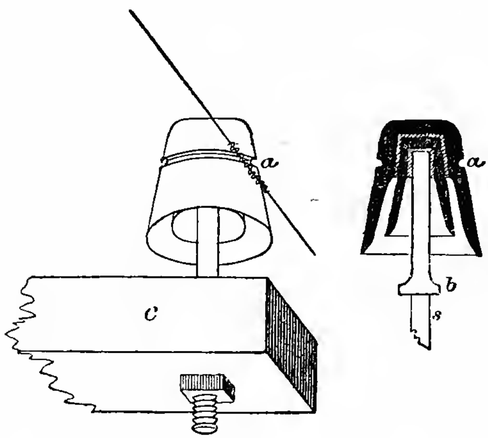


Fig. 37.

wires, or on poles along the streets, as with arc lighting, telephone, and telegraph wires in the provinces. At every post the line wire is bound by wire to a porcelain insulator at the groove marked *a* in Figs. 37 and 38, which exhibit two most approved forms now largely in use. Mr. Latimer Clark's double-cup form, shown in Fig. 37, presents a large insulating surface between the wire at *a* and the iron stalk *s*, for any leakage of current from the wire down the post to earth must occur over the two outer and two inner surfaces of the cups before the stalk is reached. Messrs. Johnson and

Phillips's fluid insulator is shown in Fig. 38, in section, and as attached to the cross arm, C, of a post. It will be seen that the

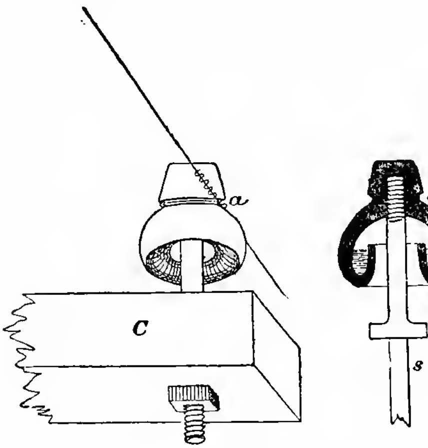


FIG. 38.

lip of the insulator is turned up at the lower end, forming a well

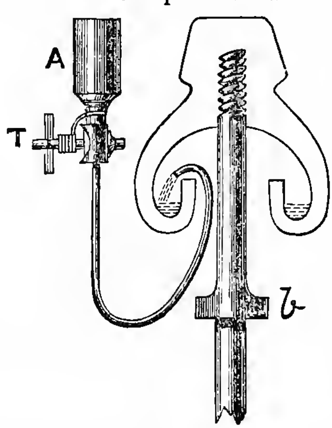


FIG. 39.

which is nearly filled with insulating oil, any leakage of current, therefore, having to traverse the outside and inside surfaces of

the porcelain cup and over the surface of the fluid. The well of this insulator is filled with a measured quantity of oil by the siphon shown in Fig. 39, which is manipulated by first filling the siphon tube with oil, then closing the tap T and filling the reservoir A. This reservoir then contains exactly the proper quantity of oil, which is delivered into the well of the insulator when held as shown in the figure, and the tap T opened. The siphon tube does not, of course, require refilling, and the next is proceeded with by closing T and filling A as before. This form of insulator offers a high insulation resistance to leakage, and is specially suitable for

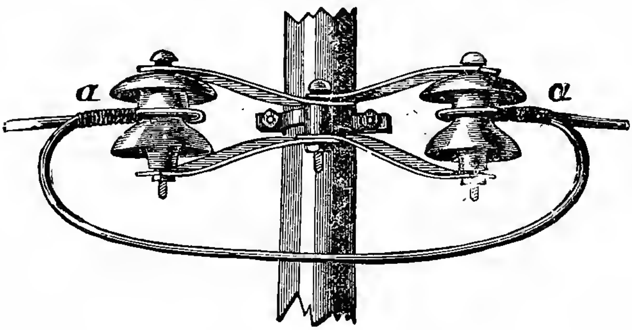


FIG. 40.

electric lighting circuits where the high E.M.F. in use tends to open up the weak points in the insulation. These insulators are fixed up by passing the stalk through a hole bored in the cross-arm C of the post, as far as the projection *b*, the nuts then being screwed on underneath, fixing them firmly in position. To avoid the strain on the insulator when the wire to and from it forms an angle, and to overcome the friction between insulator and wire when the latter is heavy or the span between two posts is long, the *shackle* insulator is employed. A double shackle insulator, supporting an electric light insulated main wire, and bolted to an iron pole, is shown in Fig. 40. The wire itself is always continuous at the shackle, and in the case of lighting wires is firmly bound at *a a* with tarred twine

after bending once round the porcelain cup. Lighting wires in London are run in this way, shackles being fixed to every pole to bear the strain of the heavy wire, and the iron poles themselves being planted on roofs of buildings, and kept in their position by wire rope stays. In addition to shackling, the strain of heavy wires is borne by a steel wire rope suspender, S S (Fig. 41), which illustrates one method of attachment. A vulcanite chair, C, carries the

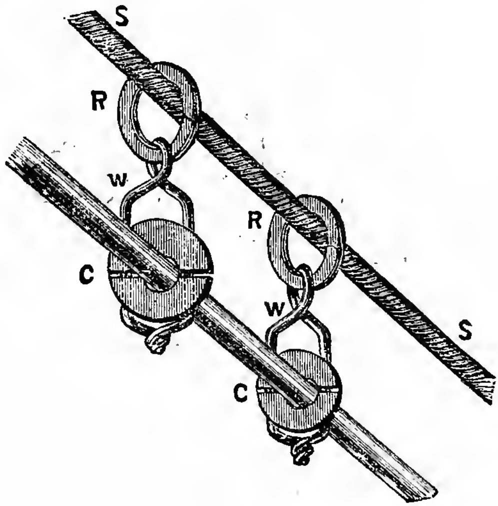


FIG. 41.

electric light wire, and this is suspended from the stranded steel wire by means of a galvanised iron wire loop, W, passed through the vulcanite chair, engaging in a split steel ring, R, which is threaded on the steel wire. The "shackling off" of a telegraph or telephone wire is somewhat different. About 2ft. of line wire is bound to each porcelain cup first; the line wire is then soldered to one of these ends, and brought round to the other end and soldered and carried on, slack being left between the two joints.

The insulation resistance of each porcelain insulator is tested under rigorous conditions after manufacture, being allowed to soak in water for forty-eight hours previous to testing, the resistance then averaging from 500,000 to four million megohms each (Prof. Ayrton in "Practical Electricity"), or say, on an average, one million megohms per insulator. When put up on a line and subjected to rain and dust the insulation resistance is, of course, reduced. In fine dry weather an insulation resistance up to 14 megohms per mile is obtained, but in wet weather it may fall to about $\frac{1}{50}$th of this; in any case it should not fall below 200,000 ohms per mile of line for telegraph and telephone work. In electric lighting by arc lamps in series with *direct* currents, or by incandescent lamps with *alternating* currents, and *transformers*, if the wires are run overhead in the above manner the insulation falls to a lower figure when *working*, owing to the high E.M.F. worked with, which tends to open up weak places in the insulating supports. For all lighting circuits, however, the wire is itself insulated and then wrapped round with a serving of tape. An arc light circuit will offer from one to two hundred thousand ohms insulation resistance per mile. Taking a line offering 200,000 ohms per mile, each resistance to leakage of 200,000 ohms is in *multiple arc* or *parallel circuit* (para. 41) with the others. Two miles would therefore measure 100,000 ohms, ten miles 20,000, and so on, and, the resistance being uniform, the rule is, divide the resistance per mile by the number of miles to obtain the total insulation resistance of the line.

It will be seen, therefore, that if even such a short length as one mile of overhead wire is taken, an insulation test with a delicate horizontal indicator as described above would readily show whether the insulation was up to or below its required value; but with a longer line, the insulation resistance being reduced, the instrument would more sensitively detect changes. The writer has seen satisfactory tests taken this way with a similar instrument on an arc lighting circuit, the more delicate instruments not being always available or practicable to use.

77. Practical Directions for taking Insulation Test.—The manner of working out a simple insulation test with an indicator as above described will now be shown. It will be noticed that no mention has been made of the testing of the insulation resistance of a submarine cable by this means, since it requires a more delicate instrument to do this, not only on account of the far higher insulation resistances of cables for the same length, but also because it is necessary to watch their behaviour during a test, and therefore a delicately suspended needle which will detect minute changes is necessary. In addition to this there is a condenser-like action causing the resistance to increase during the connection of the testing battery. This effect, observed in cables when one end is "free" and the other connected to a battery, is known as "electrification," and will be considered further when the more delicate instruments applicable to cable tests have been examined. For the present test, however, we are considering overhead lines, such as bare telegraph wires with an insulation ranging from, say, half a megohm to 10 megohms per mile, according to the state of the weather, and long overhead covered main wires supplying lighting currents from a central station where an insulation of about one to two hundred thousand ohms per mile may be expected.

Suppose now, that a sensitive horizontal indicator is connected up to one end of an overhead line 10 miles long, and that the other end of the line has been carefully "freed." Before connecting up the battery the indicator must be "set" by turning it round till the pointer is at 0°; the needle is then parallel to the plane of the coils. While doing this, everything in the immediate neighbourhood of the instrument likely to be magnetised and moved about must be removed altogether and placed at a distance where it cannot affect the needle, otherwise very discordant results may be obtained. Especially must pockets be ransacked of everything magnetised, as these approach very near the instrument when bending over to read the deflections. Anyone who is much in the neighbourhood of dynamo machines has everything about him of the nature of steel magnetised. Some little time ago a letter appeared in

one of the electrical papers from an electrician who had wasted a lot of time trying to find out what affected the needle of his testing instrument, and as he had discovered the cause of these erratic movements he kindly gave others the necessary warning. After ransacking pockets and removing everything he considered magnetic, together with his umbrella, to a distance, he at last found that his hat contained something magnetic and was doing the mischief. This he did not discover by pulling the hat to pieces, but by taking it off before the instrument and moving it about near the needle, when the effect was evident.

The circuit should now be completed by connecting the remaining terminal of the instrument to a battery of, say, ten cells, and the other pole of this battery to earth, as shown in section 76 (Fig. 36). The current now passing through the instrument is due to leakage from the line to earth, generally a uniform leakage at each post or support, and the exact resistance which this represents can be calculated by observing what deflection is produced by the same battery through the instrument and a *known* resistance. The calibration of indicators or the comparison between their deflections and the currents producing those deflections not having yet been explained, it is not supposed that we know whether the instrument reads proportionally to the current or not, in which case the known resistance would be one that could be adjusted to any amount, and this would be varied until the instrument showed the same deflection as was observed when in connection with the line. We should know then that the insulation resistance of the line was equal to the known resistance through which (together with the instrument in circuit) the same battery produced the same current. The insulation resistance per mile would then be the total resistance of the whole line multiplied by the number of miles, which in this case we assumed to be ten.

78. Insulation Resistance per Mile.—It must, however, be pointed out that this rule for calculating the insulation resistance per mile by multiplying the total resistance by

the number of miles can only be applied with accuracy to cases where the total insulation resistance of the line is very high compared to the resistance of the conducting wire itself, for when this is the case the leakage of current from the line is practically uniform; but where the conductive and insulation resistances approach each other in value

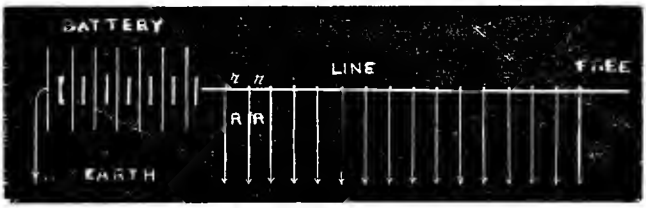


Fig. 42.

the leakage of current will be greatest at the battery end of the line and become gradually less and less along the length of the line, the effect being easily understood by considering the line as made up of a very large number of equal sections, the conductor resistance of each section being r ohms, and its insulation resistance R ohms (Fig. 42). This will be precisely

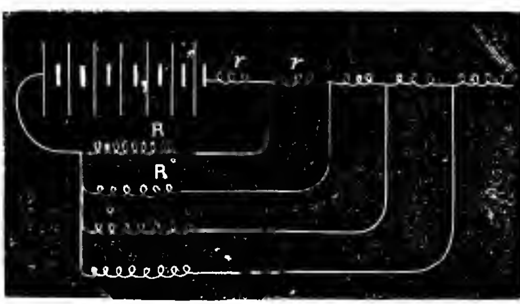


Fig. 43.

the same state of affairs electrically as the diagram in Fig. 43—that is, the resistances of the sections r and R are connected up to the battery in exactly the same way. Now it is clear that if the several resistances r r are negligibly small in comparison to the insulation resistances R R, the latter are all virtually connected direct on to the battery terminals, in

which case the current through each would be the same—that is, the leakage would be uniform along the line; but if the resistances $r\ r$ are so high, or R R are so low, that the former cannot relatively be neglected, then each resistance R receives less current as it is further removed from the battery, not only because it has an extra resistance r added to it each time, but because the potential difference acting over $R+r$ is reduced at each successive connection. In fact, the first section, $R+r$, is direct on the battery terminals, and therefore receives the highest potential difference, the next section, $R+r$, is connected to R of the previous section, instead of to the battery, hence it must receive less potential difference, and carry less current; and so on through all the sections. We should therefore expect to get less deflection for the same insulation resistance when the conductor resistance approached in value the former than when it was negligible. This is the case, and the insulation resistance therefore comes out too high, and necessitates a correction being applied. Mr. H. R. Kempe, in his "Handbook of Electrical Testing," has worked out in detail the mathematical process in evolving a correction for the above. This correction is applied as follows:—A test is taken of the resistance of the line when the far end is put to earth, which gives the *observed* total resistance of conductor. Now, instead of multiplying the total insulation resistance by the *actual* known length of the line in miles to derive the insulation per mile, the total resistance is multiplied by the number of miles as *calculated electrically* from the above test, the calculated mileage being the

$$\frac{\text{observed resistance of conductor}}{\text{known resistance of conductor per mile}}.$$

For instance, take 300 miles of ordinary telegraph line run with No. 8 galvanised iron wire, known to have a resistance of 14 ohms to the mile, and suppose that on testing this length of line its insulation was found to be 10,000 ohms, and that on earthing the other end and testing for conductor resistance this was found to be 3,500 ohms. Here the two resistances approach each other in value—that is, one cannot be neg-

lected—and therefore the true insulation resistance per mile is 10,000 ohms multiplied by the *calculated mileage*, this latter being $\frac{3500}{14} = 250$ miles. This gives the true insulation per mile to be $10,000 \times 250 = 2,500,000$, or $2\frac{1}{2}$ megohms. If calculated by the ordinary way, the result would have been $10,000 \times 300 = 3$ megohms, introducing an error of 20 per cent. The insulation and conductor resistance will approach each other either when the insulation of the line is impaired or a very long length of line is tested. The writer has had occasion to apply the foregoing correction when measuring the insulation resistance of a submerged cable, 1,400 knots in length, in which the total insulation resistance of this great length approached within four or five times the resistance of the conductor. Hence the resistance of the latter was by no means negligible in comparison.

79. **Allowance for Leading Wires in Insulation Test.**—In an insulation test of any line the line wire itself will not be connected to the instrument direct, but generally through a "leading in" wire. As the line is tested through this wire, it is important to see that it is itself well insulated. To do this the end of the leading wire affixed to the line should be disconnected or "freed" for a moment or two while the battery is put on as before and it is observed whether the indicator shows any deflection; if it does, it shows there is a leakage between the leading wire and earth. Since this would falsify the results when the line is tested, allowance must be made for it in the following manner:—Measure the insulation resistance between the leading wire and earth exactly in the manner described for a line. Now, on connecting up the line and measuring its insulation it is evident that instead of obtaining the true insulation of the line we measure the *joint* resistance of the insulation of line and leading wire because these two resistances are connected together in parallel arc. The first test measures the resistance of the leading wire alone, and the second that of the line and leading wire together. The little calculation to be made in order to derive the true

resistance of the line from these two measurements will be easily seen by reference to the conductivity of the respective circuits. It will be remembered that the term conductivity is used to express exactly the opposite property of resistance, Sir William Thomson having introduced the term "mho" for the unit of conductivity, which is taken as the exact reciprocal of one "ohm," and, in fact, is that word (or rather name) spelt backwards. The term, as far as use is concerned, is not in everyone's mouth just yet, the idea of resistance having such a deeply-rooted hold of us all; but the *idea* of conductivity lends itself very conveniently to the consideration of resistances in parallel, for we know that in this combination the total conductivity is simply the sum of the conductivity of each, and the reciprocal of the total conductivity gives the joint resistance of the whole. In the above case we require to know the resistance of the line alone. Now we can easily find the *conductivity* of the line alone by subtracting the conductivity of the leading wire from that of the leading wire and line. The resistance of the line alone is then found by taking the reciprocal of its conductivity.

Example:—A line wire is connected by a leading wire to a testing room, its other end being "freed," and a test for insulation gives 350,000 ohms; the leading wire is then disconnected where it joins the line, and a second test gives one megohm (10^6 ohms). What is the insulation resistance of the line?

Conductivity of line and leading wire $= \dfrac{1}{\cdot 35 \times 10^6}$ mhos.

Conductivity of leading wire alone $= \dfrac{1}{10^6}$,,

Therefore, conductivity of line alone $= \dfrac{1}{\cdot 35 \times 10^6} - \dfrac{1}{10^6}$,,

$$= \frac{\cdot 65}{\cdot 35 \times 10^6} = \frac{13}{7 \times 10^6} \text{ ,,}$$

and the resistance of the line is the reciprocal of its conductivity—viz., $\dfrac{7 \times 10^6}{13} = 538{,}000$ ohms.

This is known as the "*reproduced deflection*" method of measuring a resistance. In the case of insulation resistances, as above, it necessitates the use of high resistances to reproduce the same deflection, which are not always available. We are not, however, bound to reproduce the same deflection if the relation between the deflections of the instrument and the currents producing those deflections is known. The insulation test will therefore be reconsidered further on in these Papers after the above point and the employment of shunts have been discussed.

80. G.P.O. Standard Indicator.—The points in which improvements have been attempted in the construction of the current indicator or detector have been chiefly the following:—

1. Reduction of the friction on the moving needle to a minimum, doing away with irregularities in the relation between currents and deflections, due to variation of friction, and rendering the instrument more sensitive.

2. Protection of bearings and needle spindle points or pivot points while the instrument is subjected to shaking and otherwise knocking about in transit.

3. Best method of winding the coil to give maximum effect when occupying a given volume.

4. Facility of reading quickly and accurately the deflections.

5. Easy accessibility of all parts of the apparatus for purposes of cleaning or adjustment, if required.

6. Sensitiveness, or variation of deflection for slight changes in the current.

An instrument combining portability with the above conditions being a great acquisition to the ordinary battery and line testing at the General Post Office, a great deal of attention has been brought to bear upon its improvement by Mr. Preece and Mr. Kempe, in conjunction with Mr. Eden, with the result that a "standard" form has now been evolved as the "survival of the fittest" of all their experiments and

trials. This instrument is illustrated in plan in Fig. 44, and

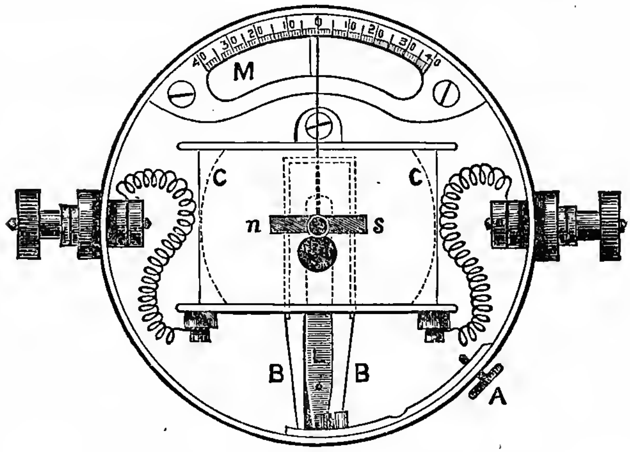


FIG. 44.

in elevation in Fig. 45, the cover in this figure being removed

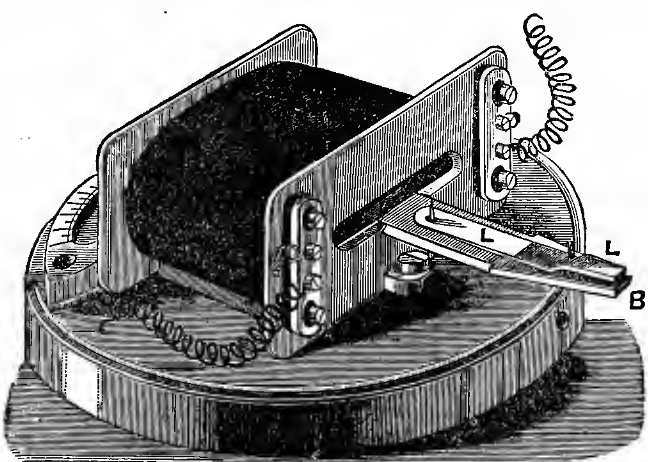


FIG. 45.

and the needle support withdrawn from inside the coil. The

needle itself is shown in Fig. 46, and is a very light steel magnet of the shape *n s*, to which is affixed a brass cap, C, holding at its centre the sapphire cap which rests on the pivot point when the needle is in position. The pointer *p*, at the extremity of which the readings are taken, is a rigid and light strip of ebonite fixed at right angles to the needle, and the weight *w* of the same material serves to balance the needle in a horizontal position. In both the other figures the movable brass slide B, on which is fixed the upright steel pivot to carry the needle, is shown. This slide containing the needle is easily inserted into or withdrawn from the coil, its movement being kept rigidly in one direction by its bevelled edges sliding in guide grooves cut along the interior of the brass bobbin. A

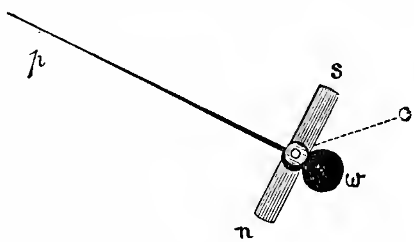


Fig. 46.

slight projection on the slide seen in the figure fixes the limit to which the slide can be pushed into the coil, and insures the needle always being in the centre of the coil. The steel pivot point is finished with great care, being carefully examined under the microscope during manufacture, the sides converging towards the point in as nearly as possible the curve of a parabola. Protection of the point when the instrument is not in use is secured by raising the needle from off it by means of the lever L L. It will be easily seen that by depressing the short end of this lever the other end lifts the needle off the pivot, and it is arranged that the downward play of the short end (effected by moving the screw head A sideways to the left) shall be just sufficient to raise the needle through the requisite distance to bring its cap into close contact with the top

of the interior of the brass bobbin, thus preserving it completely from movement during transit. The dotted lines in the centre of the coil in Fig. 44 show the needle mounted in position. Its point then rests over the divided scale, attached to which is a mirror, M, the purpose of which in instruments of this class is to avoid the error of "parallax" while reading the deflections. This error will always be more or less introduced according as the point of the needle is at a greater or less distance from the scale when the direction of the line of sight from the eye of the observer to the needle is not always constant. By means of the mirror (attached to all accurate instruments), the reflected image of the needle is visible in the mirror, and the reading on the scale is not taken until the needle is seen to exactly cover its own reflection therein. In order to throw as much light as possible on the scale and needle, a glass window is let into the side of the brass cover opposite the scale, and the top of the cover being of glass, there is every possible facility for very accurate readings. It is seen, therefore, that conditions 1, 2, and 4 are complied with. The third condition is carried out by Mr. C. J. Simmons, of London, the maker of the instrument, by "grading" the wire composing the coil, and giving the outside contour of the coil the shape of the curve indicated by the dotted lines *c c*. When the size of the coil—that is, the space to be occupied by the wire—and its required resistance have been once fixed upon, it has been pointed out by Sir William Thomson that the maximum magnetic effect, or, what is the same thing with the same current strength, the maximum number of *turns* of wire, can be wound into the allotted space when the wire is graded in sections of different diameter. That is, the winding is commenced nearest to the needle by fine wire, a certain portion of the available space being filled up with this winding. Then a wire of larger diameter is jointed on to this and the winding continued. This is done for three or four different sizes of wire, all being calculated at first so as to add up to the required resistance when completely wound on. In this instrument the wire is graded three times, the final resistance being 800 ohms. The shape of the exterior of the

coil is also a matter of consideration, and the same authority gives it as the most effective winding for the coil of a current indicator or galvanometer when the greatest number of turns of the wire are wound near to the needle when at rest—that is, at right angles to the axis of the coil—the shape of the curve itself being accurately expressed for coils whose turns of wire are in circles by a mathematical equation.

The fifth condition is ensured very effectively as regards the accessibility of the needle, this having already been shown. The ends of the coil go to separate terminals on an insulated brass plate, two other terminals on which serve to connect a short spiral of thick wire to the instrument terminals, in the usual way. The finish of the instrument, as turned out by Mr. Simmons, is excellent, the G.P.O. having relegated all their old astatic forms of indicator to this maker to convert into the "approved converted form," with the parts somewhat similar to the above. The instrument which has been described above is not "astatic." An instrument is said to be astatic when it is provided with two parallel needles mounted on one spindle, and turning together, one being inside the coil and the other outside, and the opposite poles of the two needles pointing the same way. By this device the controlling force of the earth's magnetic field upon the needle, which tends to draw it towards the magnetic meridian when it is deflected away from it by a current, is almost entirely done away with, only sufficient difference between the magnetic moments of the two needles existing necessarily in order to keep the needles at zero when no current is passing through the instrument coil. The writer has taken a "calibration curve" of the instrument described above, and finds that there is a "straight line law," or proportionality, between currents and corresponding deflections up to an angle of 27deg., this being nearly the whole length of movement of the needle, which, owing to the construction of the coil, is limited in its movement to 35deg. In the following chapter the calibration of current indicators will be considered, with the plotting of curves exhibiting the relations between the strength of the current and the angular movement of the needle.

CHAPTER VIII.

THE CALIBRATION OF CURRENT INDICATORS.

81. Simple Method of Calibration by Low-Resistance Cells.—The relation between the angular deflections of a needle and the currents causing those deflections, in an indicator, is a most useful thing to be known about the instrument. The currents causing given deflections on any instrument may be directly proportional to those deflections, or they may vary as some known function of the angles of deflections, or there may be no settled relation between them at all. It is in the latter case that "calibration" is desirable, and the relation between the angular deflections and currents is generally shown by means of a curve, which can be referred to when it is required to know what is the strength of current corresponding to a given deflection. There are different methods of carrying out the calibration of an indicator, more or less suitable according to the resistance and sensibility of the instrument. We shall first take the simplest method, which can readily be applied to the calibration of high-resistance instruments when a few similar *low-resistance* cells are available. The internal resistance of a cell will be low when its plates are of large size and fixed near together. Almost any type of cell will do for this test, provided its plates are of large area, in addition to which the cells should not polarise; this latter difficulty, however, being very unlikely to occur, as the time of contact need only be very short to take a reading of the deflection, and the comparative high resistance of the instrument would allow very little current to pass from the cells. The

Thomson tray cells (Fig. 47) previously described in these papers (para. 18) are of very low internal resistance, varying from $\frac{1}{10}$th to $\frac{1}{2}$ an ohm per cell, and would be very suitable; or if some charged accumulators were available, these would do better still, their resistance being only about $\frac{1}{50}$th of an ohm per cell.

Taking, in the first instance, a vertical detector of high resistance compared to the cells, it would be first connected to one cell in the manner shown in Fig. 47; the deflection might

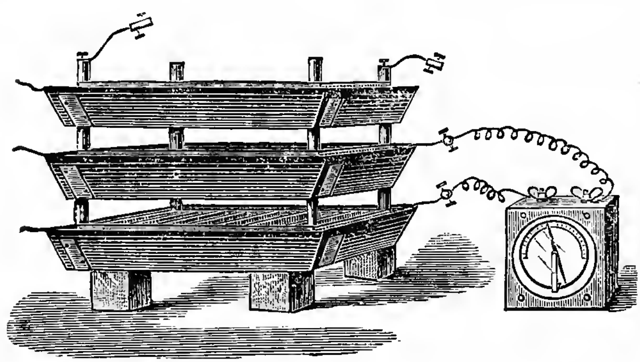


Fig. 47.

then be large or small according to the weight of the needle. If the deflection is only a few degrees, we should proceed, after noting it down, to connect up two cells, then three, and so on till the needle was deflected to near the end of the scale, say to 85 degrees, noting down opposite each number of cells the corresponding number of degrees deflection. Now the currents are very easily calculated, each cell having a fixed E.M.F., and its internal resistance so low in comparison to that of the instrument that it may be neglected. An example of this kind actually taken of a vertical detector of 220 ohms resistance is here appended. The cells used were secondaries,

or accumulators, measuring 0·02 ohm and 2 volts per cell, and the readings taken were as follows:—

No. of Cells.	Deflection.	Volts at Terminals of Instrument.
1	10	2
2	42	4
3	64	6
4	75	8
5	81	10
6	85	12
7	87	14

Now, it is desirable to obtain some intermediate readings between 10 and 42 degrees and below 10 degrees, as a very wide difference in the deflections is noticed between one and three cells. We cannot get these intermediate readings by the cells alone, but must add a resistance to the detector to reduce the difference of potential at its terminals. First, it must be shown that the number of volts at the instrument terminals when connected direct to the cells is practically equal to the E.M.F. of the cells. By Ohm's law we have the current passing through the instrument is equal to the

$$\frac{\text{volts at instrument terminals}}{\text{resistance of instrument}}$$

and therefore the number of volts at the instrument terminals is always the product of the current and the instrument resistance. But the current is also, by Ohm's law, the

$$\frac{\text{total E.M.F. of battery}}{\text{total resistance in circuit}};$$

therefore, the number of volts at the instrument terminals is equal to the

$$\text{total E.M.F. of battery} \times \frac{\text{resistance of instrument}}{\text{total resistance in circuit}}.$$

Now, if the instrument was connected directly to, say, 5 cells, the E.M.F. of battery would be $5 \times 2 = 10$ volts, and

its resistance $0{\cdot}02 \times 5 = \frac{1}{10}$ ohm; and therefore the number of volts at the instrument terminals would be

$$10 \times \frac{220}{220\frac{1}{10}} = 10 \times \frac{2200}{2201},$$

where it is evident that the fraction by which the E.M.F. of 10 volts is multiplied is so nearly unity, that we practically have the same number of volts at the instrument terminals as we have E.M.F. in the battery. The reason is that the internal resistance of $\frac{1}{10}$th ohm is so small in comparison to 220 ohms, that it causes an inappreciably small loss of E.M.F. in the battery. This accounts for the third column given in volts. Now, to get the intermediate deflections in this case, extra resistances were added to the instrument, so as to produce respectively 1, $1\frac{1}{2}$, 3, and $4\frac{1}{2}$ volts at the instrument terminals. By taking one cell, and adding a resistance equal to the instrument, the potential difference at the terminals became

$$2 \times \frac{220}{220 + 220 + {\cdot}02} = 2 \times \frac{1}{2} = 1 \text{ volt};$$

and by making the extra resistance equal to one-third that of the instrument ($= 73{\cdot}3$ ohms) the volts at the terminals became

$$2 \times \frac{220}{73{\cdot}3 + 220 + {\cdot}02} = 2 \times \frac{3}{4} = 1\tfrac{1}{2} \text{ volt.}$$

Similarly, to obtain 3 volts, two cells were connected, and the extra resistance made the same as in the previous case. This gave

$$4 \times \frac{3}{4} = 3 \text{ volts.}$$

For $4\frac{1}{2}$ volts at the terminals, 3 cells were joined up, the added resistance being the same as before, which gave

$$6 \times \frac{3}{4} = 4\tfrac{1}{2} \text{ volts.}$$

In any case it will be seen from the above that to obtain any given number of volts at the instrument terminals when

low-resistance batteries are used as above, a resistance must be added equal to the resistance of the instrument multiplied by the following fraction—

$$\left(\frac{\text{E.M.F of battery}}{\text{number of volts required}} - 1\right)$$

The intermediate deflections observed with the above were then as follows :—

No. of Cells.	Added Resistance.	Deflection.	Volts at Instrument Terminals.
1	220	4	1
1	73	7	$1\frac{1}{2}$
2	73	22	3
3	73	48	$4\frac{1}{2}$

thus making eleven observations altogether, from which the curve in Fig. 48 is plotted. The angles of deflection are marked on the vertical ordinate, and the potential difference (P.D.) in volts at the terminals on the horizontal ordinate. The latter ordinate is also marked in units of current (milli-amperes), the current in these units being simply the

$$\frac{\text{number of volts at terminals}}{\text{resistance of instrument}} \times 1000,$$

one milliampere being the $\frac{1}{1000}$th part of an ampere.

Now, since the denominator in the above is constant, it is clear that the currents through the instrument are proportional to the volts at the terminals, and therefore the calibration curve connecting deflections with volts at the terminals is precisely the same as that connecting deflections and currents.

82. **Use of the Curve.**—It will be noticed that the instrument is very unsensitive above 70 degrees and below 10 degrees—that is, considerable change in the current strength is necessary to produce appreciable change in the deflection; but at angles between 20 and 50 degrees the instrument is most sensitive. The steeper the incline of the curve, the

more marked will be a change of deflection for a change of current.

The value of any current may now be found by reference to the curve ; thus, if 60 degrees deflection was observed the corresponding current could be found by placing the edge of a flat ruler at number 60 on the vertical degree ordinate, and the edge of the rule being horizontal would cut the curve in some point. Marking this point lightly with a pencil the rule would be turned vertically in a line from the point on the

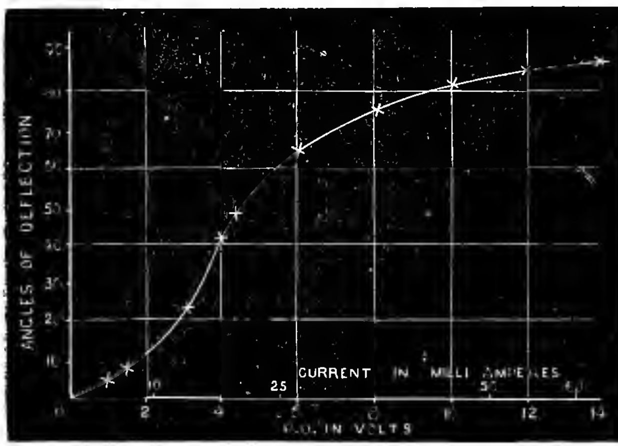


Fig. 48.

curve to the horizontal ordinate. This would cut the latter at 25 milliamperes, which is the required current. It should be mentioned here that practical working curves are usually about four or five times the size of the one in the figure ; in fact, the larger the scale they are drawn to and the more observations taken from which to plot them the better.

83. Proportionality between Currents and Deflections.— This curve is seen to exhibit no "straight line law" or proportionality between currents and deflections except, perhaps, up to 10 degrees—that is, every scale degree movement of

the needle does not represent an equal increase of current. As simple examples of proportionality between indications and scale degrees, there might be cited the Salter's balance, where every scale degree movement of the pointer means an equal increase of weight suspended to it; or the steam-pressure gauge, in which every equal scale division means equal increase in steam pressure; or the thermometer, whose scale divisions are equal, and mean equal rises in temperature; or the ordinary clock, whose dial is divided into twelve equal parts, each of which mean equal intervals of time. If we were to plot the relation between degrees deflection, and weights in the spring balance, or between divisions deflection, and pressure in the steam gauge, or between divisions rise of mercury, and temperature in the thermometer, or between movements of clock hand and intervals of time in the ordinary clock, precisely in the same manner as the readings are plotted for the detector in Fig. 48 *ante*, we should get a straight line instead of a curve, showing absolute proportionality between the indications and degrees of scale. This straight line law or absolute proportionality is very desirable in instruments used in the measurement of large currents used in electric lighting, and has been very successfully carried out by Profs. Ayrton and Perry in their ammeters, or current measurers, in which the divisions through which the needle is deflected are made equal to each other and correspond to equal increments in the current strength throughout the whole of the scale. Messrs. Walmesly and Mather have devised a galvanometer whose deflections follow the proportional law up to about 50 degrees. These instruments will be referred to again in detail, when the different kinds of galvanometers are examined.

84. **The Horizontal Astatic Indicator.**—If, now, the indicator to be calibrated is of the horizontal type, we should expect to find the needle move round a large number of degrees for a very small current, since the horizontal mounting of the needle, whether by a pivot or by a silk fibre suspension, admits of greater sensibility. One or two cells would then furnish the

maximum current required, the largest deflection being obtained with the battery connected direct to the instrument, and the deflection being gradually reduced by the addition of extra resistance to the circuit. We shall in this example also select cells (such as secondaries or large size primary cells) of low resistance, negligible in comparison to the resistance of the instrument coils.

A curve taken in this way of the astatic instrument shown in Fig. 49 is here appended. The two needles marked *n s* in reverse ways may be seen in the figure, rigidly connected together, and the pair suspended by a silk fibre, one needle being inside and the other outside the coils forming the astatic system. To set the instrument it must be turned round till the needles are parallel to the plane of the coils, which will be the case when the top needle points to zero on the graduated dial D. This can be very conveniently effected without shifting the base of the instrument when once levelled by moving the coils alone, these being attached to a hollow wooden base, B, capable of being turned about the centre by means of the handle A through an angle of about 60 degrees. The base B contains a sufficient length of flexible wire, spirally wound, to admit of this movement of the coil while it is connected permanently to the two terminals. It is important to see that the lower needle does not touch the interior of the bobbin during any part of its movement, and once the needles are perfectly still, this can be tested very well by moving the handle A right and left through its full range, which should not cause the slightest movement of the needles. If it does, the interior of the bobbin should be looked through to see if the lower needle is too high or too low, and its height can then be adjusted as required by raising or lowering (not screwing round) the suspension head E, to which the silk fibre is attached.

85. **Connections for Calibrating.**—The instrument, after being "set" as described, should be connected to a box of standard resistance coils A, and a low-resistance cell, as shown in Fig. 50. The higher the resistance of the coils in the instrument the more margin may be allowed for the resist-

ance of the cell while considering the latter as a negligible quantity—that is, if the coils were, say, 1,000 ohms, as

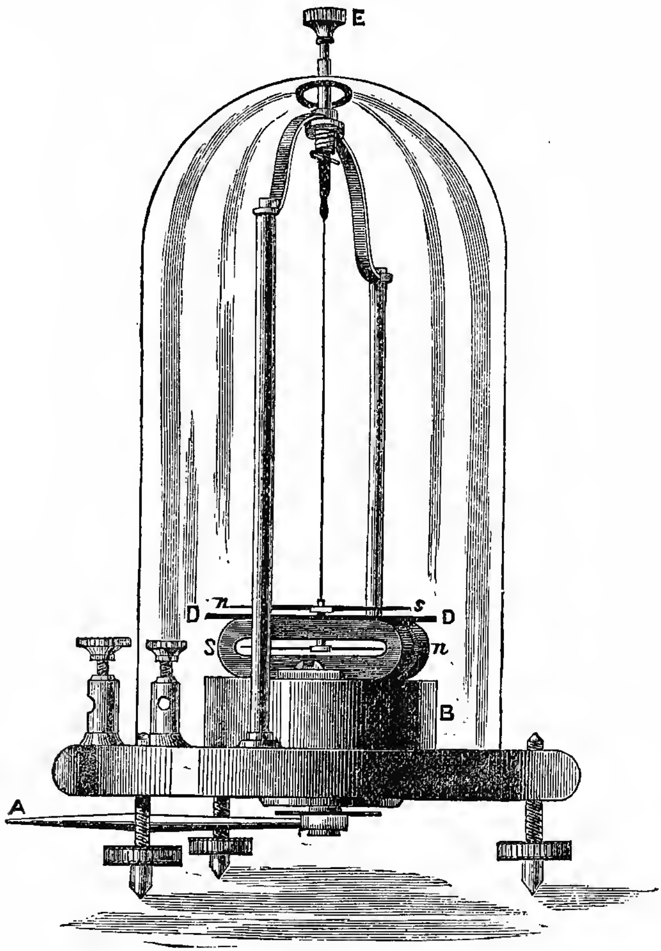


FIG. 49.

high a resistance as 3 or 4 ohms in the cell would be practicably negligible. Horizontal instruments are usually

wound to resistances from 500 to 1,000 ohms. If there is no very low-resistance cell available, such as a primary battery with large plate surface in use for lighting, or a charged accumulator, or a large size Daniell or Fuller cell, three or four cells may be connected together in multiple arc —that is, all the zincs together to form one terminal of the

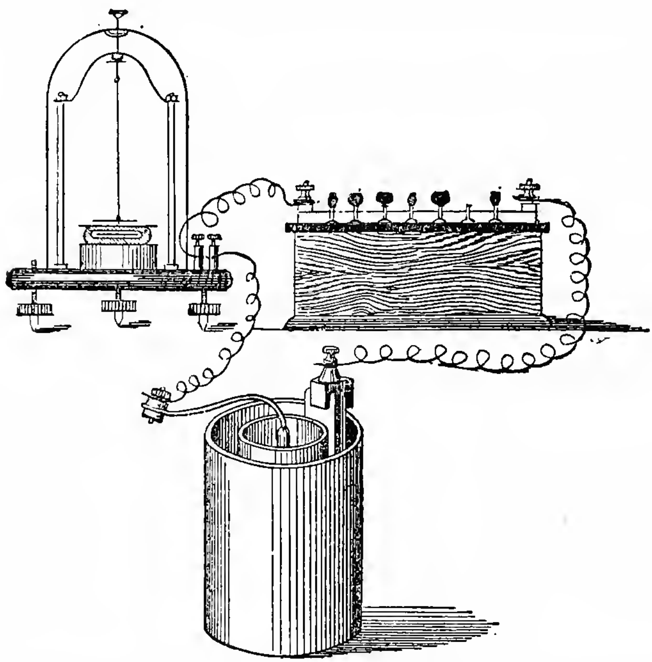


Fig. 50.

battery, and all the coppers or carbons together to form the other. This will then act as one cell of as many times the amount of plate surface as there are cells connected up, and the internal resistance will be diminished in proportion to the number. Thus, four cells in multiple arc would have one-fourth the internal resistance of one cell, five cells one-fifth, and so on, while the E.M.F. of the combination,

in this manner, of any number of similar cells would be the same as that of one.

86. Construction of the Adjustable Resistance Box.—The resistance-box is very simply manipulated for altering the

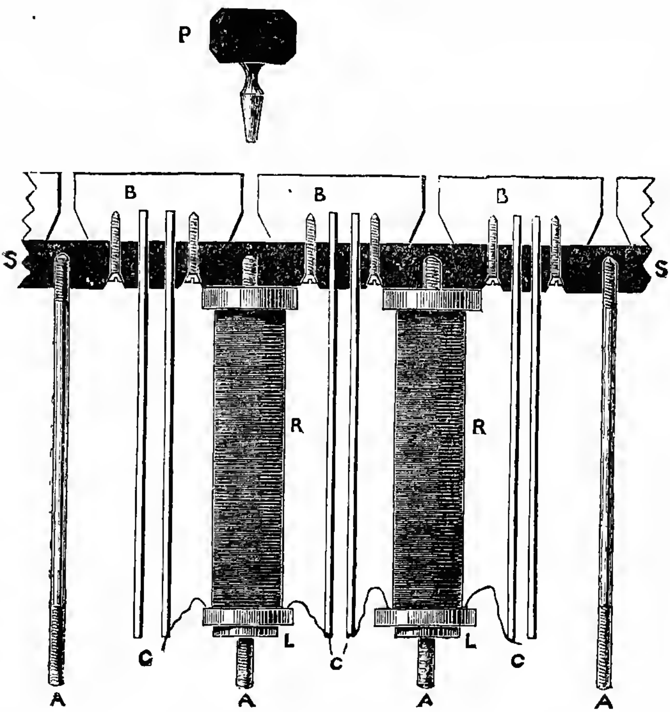


Fig. 51.

amount of resistance in circuit. A vulcanite-headed brass plug P (Fig. 51), when inserted into a round hole between two adjacent brass blocks, B B, short-circuits, or metallically bridges across, the resistance coil R, whose ends are connected to those two blocks. A number of coils wound accurately to different fixed resistances at the temperature *supposed* to be

generally prevalent in this climate, viz., 60°F., are contained in the wooden box, the bobbins of the coils being fixed on the under surface of the ebonite slab S S, which forms the lid of the box. These wooden bobbins when wound are slipped over the brass rods A A, which are screw-threaded at each end, serving to screw them into the ebonite slab at one end and to have a brass lock nut, L, screwed on at the other end after the bobbin is slipped on. The copper wires C C are soldered to their respective brass blocks, and it is to these wires that the ends of the coils are soldered, the final adjustment of the resistance of a coil being made at one of these joints. There are two of these wires to each brass block, so that each coil is attached to its own individual pair of copper wires, and the resistance of each coil can be finally adjusted separately. The final resistance, then, between two brass blocks is that of the coil plus the two copper wires which connect its ends to the blocks; and by having a separate pair of copper wires to each coil, a given resistance introduced into the circuit by unplugging *one* coil marked that amount will be exactly equal to that introduced by unplugging a number of coils the sum of whose marked resistances is equal to the given resistance. Thus we should insert exactly 100 ohms, whether the 100-ohm coil alone was unplugged or the four coils marked 50, 20, 20, and 10 were unplugged together, or, in fact, in whatever way the said resistance was made up. If there were only one copper wire connection to each brass block, a given resistance made up by unplugging a number of coils would be *less* than when only the coil marked that resistance was unplugged. The coils are generally wound with German silver wire, the resistance of this alloy (nickel one, zinc one, and copper two parts), according to Dr. Matthiessen, increasing by only $\frac{44}{1000}$ per cent. of its value for one degree centigrade rise of temperature. The more costly alloy of platinum-silver increases by only $\frac{31}{1000}$ per cent. of its value for the same rise in temperature, and therefore is more suitable for resistance coils, but is only rarely used on account of its cost. The new alloy termed platinoid (formed by the addition of one or two per cent. of metallic tungsten to German silver) only increases in resistance by $\frac{21}{1000}$ per cent.

of its value for the same rise in temperature, and has the additional advantage of measuring a little more than $1\frac{1}{2}$ times the specific resistance (para. 30) of German silver, requiring, therefore, less wire for a given resistance. It remains, however, to be ascertained whether the resistance of this alloy undergoes any permanent change after long usage. The wound coils are well baked and steeped in hot paraffin wax, melted to above the boiling point, before final adjustment. After this is accomplished the coils are steeped in the same wax, but this time only sufficiently warm to quickly coat them. The resistance of each coil is marked immediately above it on the upper surface of the ebonite slab, and the total resistance introduced into a circuit by means of the box is found by adding together all the marked numbers opposite holes where plugs have been removed. Sixteen coils are generally fitted to a resistance box, which is intended to read up to 10,000 ohms, the resistances of the sixteen coils being of such values that any number of ohms from one to 10,000 can be introduced in the circuit by arranging the plugs.

87. Method of taking Observations for the Curve.—The astatic indicator shown in the figure measured 545 ohms, and gave a deflection of 75 degrees when a resistance of 405 ohms was unplugged in the box, two cells, coupled together *in series*, being employed, each with an E.M.F. of 1·8 volt. With this deflection the current was therefore

$$\frac{(1{\cdot}8 \times 2) \text{ volts}}{(545+405) \text{ ohms}} = \frac{3{\cdot}6}{950} = {\cdot}0038 \text{ ampere},$$

or 3·8 milliamperes.

The deflections were then diminished by increasing the resistance in the box—that is, by unplugging more coils, and the corresponding deflections for each change of resistance were noted. For facility of calculation afterwards it is advisable to make the unplugged resistances in the box such that when added to the resistance of the indicator a multiple of ten is always obtained. For example, an even resistance of 2,000 ohms in the circuit is obtained when $2{,}000 - 545 = 1{,}455$

ohms is unplugged in the box, and so on. Some eleven or twelve readings were taken in this way, which are shown plotted as a curve in Fig. 52, the currents being calculated as above in milliamperes. If the only cells available were of small plate surface, and therefore high resistance, and could not conveniently be connected together in parallel arc to reduce their resistance, as might be the case with a battery of a number of cells contained in a trough, the resistance of the battery used would have to be added to

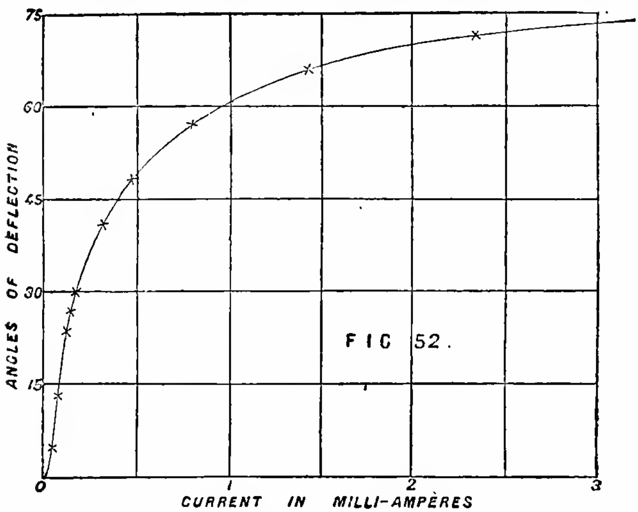


FIG 52.

the other resistances in the circuit in the calculation of the current. One or two ready methods of measuring the resistance of a battery will be considered in a later chapter.

While taking the above calibration test there is no advantage in having a "key" in the circuit, by means of which the latter can be "made" and "broken," since if the circuit is broken after each reading, the needle of the instrument is set swinging inconveniently. The circuit should be permanently connected as in Fig. 50; and it should be observed whether each deflection of the needle remains steady, since if the deflec-

tion falls slightly it shows that the battery is polarising, and the readings, consequently, valueless, because the E.M.F. of the battery is slowly diminishing. This will not be at all likely to occur, however, as the resistance of the instrument is high, and we are adding resistances to it each time a reading is taken. To be sure, however, that no polarisation has occurred, the first reading is repeated, after the whole set have been taken, and if this is precisely the same as at first, it proves that the E.M.F. of the battery has kept constant during the test.

88. Checking Oscillations of Needle.—With each change in the resistance unplugged in the box the needle will be set swinging slightly, and instead of losing time in waiting for it to come to rest, its oscillations may be checked by a small magnet held in the hand. When the needle swings towards the magnet the latter should present the same pole towards the needle, repulsion between the two like poles checking the swing of the latter. The hand holding the magnet must then be taken right away from the instrument, when the needle swings the other way, unless the operator is quick enough to present the other pole of the magnet to the needle on its receding swing. With practice the oscillations can be checked almost immediately in this way, but care must be taken that once the needle is brought to a standstill the little magnet is placed on the table at arm's length away so as not to affect the steady deflections. With even light needles a swing of a very small angle will last a tediously long time, owing to the long fine silk fibre suspension and the inertia of the moving needle, and time is saved by a little dexterity in checking the swings in the above manner.

89. Curve of G. P. O. Standard Indicator.—The relation between currents and angular deflections up to 28 degrees on the standard horizontal indicator of the General Post Office (illustrated in Fig. 45) is shown by Fig. 53. This is seen to be practically a straight line—that is, the instrument is practically a proportional one up to about 28

degrees. It should be noticed, however, that on closer examination the line is not absolutely straight, although very nearly so, but is tending to bend round slightly. This may be noticed by putting the edge of a flat rule along the line, and seeing how far the direction of the line coincides with the straight edge of the rule, which in this case exactly coincides from 0 to 15 degrees, the remainder of the line up to 28 degrees being very slightly removed in direction from

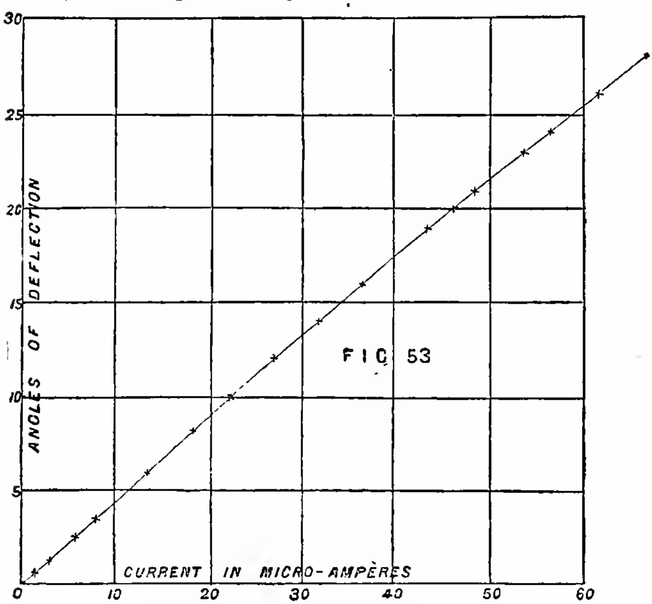


FIG. 53

this portion. The readings for this instrument were taken in the same way as the last, except that only one cell was used, the E.M.F. of which was 1·868 volt. It will also be observed that the currents, which are very small, and are measured in millionths of an ampere, or micro-amperes, produce sensibly large deflections, which means that the instrument will give perceptible indications when it is circuited with very high resistances and a low E.M.F. For instance, with the single cell used above, it was necessary to insert one megohm (one mil-

lion ohms) resistance in circuit with the instrument, in order to get the lowest deflection, viz. : about $\frac{7}{10}$ths of a degree. The current producing this deflection would be, in amperes, the

$$\frac{\text{E.M.F.}}{\text{total resistance}} = \frac{1{\cdot}868 \text{ volt}}{1 \text{ megohm} + 800 \text{ ohms}}.$$

We may, however, neglect the resistance of the instrument (800 ohms) when added to so large a resistance as one million ohms, and multiplying the current in amperes by one million to reduce it to micro-amperes, we have

$$\frac{1{\cdot}868}{10^6} \times 10^6 = 1{\cdot}8 \text{ micro-ampere,}$$

causing the above perceptible deflection. This shows that the instrument is capable of measuring very large resistances.

90. Comparison of Instruments by their Figure of Merit.—Comparing the calibration curves of these instruments, it will be seen at once that the same current produces a different deflection on each. This difference is due to the construction of the instrument, the controlling force on the needle, the kind of pivoting or suspension of the needle, and the number of turns of wire on the coils. As a basis of comparison between instruments differing in these respects, we have to determine for each instrument what is termed its *Figure of Merit*, which is simply the total amount of resistance in ohms (including the instrument coil resistance) through which, with one volt E.M.F., a deflection of 1 degree or division is produced. Now, by Ohm's law, this resistance will be equal to

$$\frac{1 \text{ volt}}{\text{current producing 1 degree deflection}},$$

which is the same thing as the *reciprocal of the current in amperes producing 1 degree deflection*, and it is by these words that the figure of merit is usually defined. Hence, an instrument with a high figure of merit is one which will show a perceptible deflection with a very small current.

The figure of merit may be found from the calibration curve by taking any current on the *first straight portion* of the curve

—that is, the portion of it which is proportional, and observing what deflection this corresponds to; then dividing the current by the number of degrees deflection we have the current for 1 degree, the reciprocal of which, after reducing to amperes, is the figure of merit. Taking Fig. 53 we find by inspection that 12 degrees deflection is produced by 27 micro-amperes; and therefore 1 degree would be produced by $\frac{27}{12}$ $= 2\frac{1}{4}$ micro-amperes, which is equal to $\frac{2\frac{1}{4}}{10^6}$ amperes.

The figure of merit is therefore the reciprocal of this current in amperes, and is

$$\frac{10^6}{2\frac{1}{4}} = 10^6 \times \frac{4}{9}.$$

Now, by inspection of the straight portion of the curve for the astatic instrument in Fig. 52, we find that 18 degrees deflection is produced by 0·1 milliampere, or 100 micro-amperes; and therefore $\frac{100}{18}$ micro-amperes, or $\frac{100}{18 \times 10^6}$ amperes, would give 1 degree deflection. The figure of merit is then the reciprocal of this, which is

$$10^6 \times \frac{18}{100}.$$

If the two instruments are now compared by their figures of merit we have

$$\frac{\text{suspended needle instrument}}{\text{pivoted needle instrument}} = \frac{10^6 \times \frac{18}{100}}{10^6 \times \frac{4}{9}} = \frac{81}{200},$$

that is, the figure of merit of the pivoted needle instrument is $2\frac{1}{2}$ times that of the other, which is the same thing as saying that the astatic instrument requires $2\frac{1}{2}$ times as much current as the pivoted needle instrument for a deflection of 1 degree. If the curve is not determined, the figure of merit may be easily found by passing a weak current through the instrument only sufficient to move the needle a few degrees, say 5; for the

reason that mostly all instruments read proportionally to the current for the first few degrees. The figure of merit is then simply *this deflection divided by the number of amperes producing it.*

91. The Simultaneous Calibration of Instruments of Similar Sensitiveness.—The calibration of two or three instruments may conveniently be effected at the same time by connecting them together in "series" (para. 40) with a cell or two and an adjustable resistance box (para. 86) in circuit. Whatever current, then, passes through one instrument passes through all, so that, proceeding in the same manner as for one instrument, and supposing, in the first instance, that the instruments are all of the same type, we should expect to obtain somewhere about the same deflection on each for every current passed through.

The number of cells would be arranged to give a large deflection on each indicator when no resistance was unplugged in the box, and the successive readings would then be taken by increasing the resistance unplugged, until the deflections became very small.

The current for every change in the resistance in circuit would be, by Ohm's law, the

$$\frac{\text{E.M.F. of battery}}{\text{total resistance in circuit}},$$

where the total resistance is the sum of the resistances of the coils of the instruments added to that of the cells (if appreciable) and the resistance unplugged in the box. It is advisable for facility in calculating the currents to make every resistance unplugged in the box of such an amount that, added to the resistances permanently in circuit, the total number of ohms for each current becomes a multiple of ten.

The deflections of the needles of each instrument are read off every time a change is made in the current strength, and from these readings curves are plotted for each instrument as previously described.

92. The Simultaneous Calibration of Instruments differing in Sensitiveness.—It more frequently occurs, however, in practice that the instruments are dissimilar as regards their sensitiveness. A glance at the curves of the three instruments which have already been considered will show that any given current produces different deflections on each, or, what is the same thing, different currents are required by each instrument to produce a given deflection. For example, taking 25 degrees deflection on each instrument, the curve in Fig. 48 shows that the vertical indicator requires 15 milliamperes of current for this ; the astatic instrument curve (Fig. 52) shows that 120 micro-amperes are required, and the horizontal pivoted needle instrument curve (Fig. 53) shows that about 60 micro-amperes would give this deflection. The latter instrument produces the deflection with the weakest current, and therefore is the most sensitive, the vertical detector being the least sensitive. Now, if three instruments of different degrees of senitiveness were joined together in series and a current sent through sufficient to cause a maximum deflection on the *most* sensitive instrument, the deflections on the less sensitive indicators would be too small to commence with, and therefore by some means the sensitiveness of the instruments must be equalised so as to produce about maximum deflections on each to start with.

93. Use of the "Shunt."—The sensitiveness of an indicator can be reduced by "shunting" it—that is, by connecting a resistance to its terminals ; part of the current then flows through this resistance or is "shunted" past the instrument. A box containing resistance coils in which the resistance may be adjusted to any value by means of plugs, as before described, is connected to the terminals of the most sensitive indicator. The resistance so connected is technically known as a "shunt" on the instrument, and its duty is to conduct away a portion of the current, so that only a certain part of the current, say $\frac{1}{n}$th, flows through the instrument. If, then, we observe the current flowing through the instrument so shunted, we know

that the current in the main circuit is n times as much. The value of the number "n" is then what is termed the "*multiplying power*" of the shunt.

94. Practical Case of the Application of Shunts.—A practical instance will best serve to make the action of the shunt clear. In Fig. 54, A B and C represent three indicators of different degrees of sensitiveness, which were calibrated at the same time. A few preliminary observations were taken,

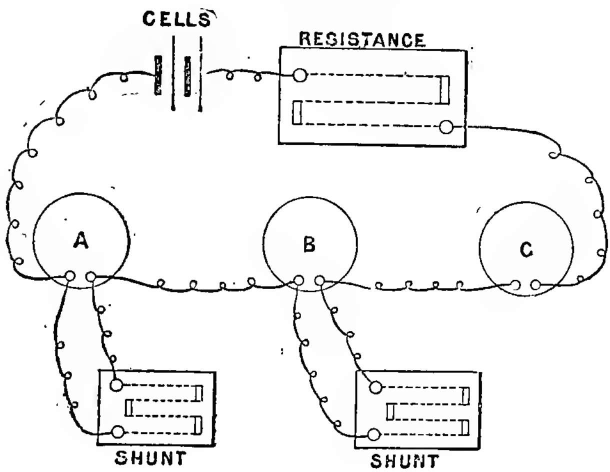


FIG. 54.

employing one secondary cell of negligible resistance and of 2 volts E.M.F. This was circuited in series with an adjustable resistance box, and the three instruments as shown. All three instruments had their needles suspended horizontally by silk fibres, but the instruments were very different in other respects. A was an astatic instrument, similar to that shown in Fig. 49, and wound to 544 ohms, B was a Walmesley-Mather instrument wound to 192 ohms, and C had a small needle surrounded by a large circular coil, constituting ordinarily a tangent galvanometer, but employed in this case with its coils moved out of the mag-

netic meridian by 60 degrees. This instrument was wound to 253 ohms, and the divided scale over which the needle could be deflected was marked out into 200 divisions. Before any instrument was shunted, it was found that with one cell in circuit A was deflected 80 degrees, B 52 degrees, and C 74 degrees, 11 ohms being unplugged in the resistance box, which, added to the total resistance of the instruments, made up the entire resistance of the circuit to an even 1,000 ohms. The deflection on C, which should be nearly 200 divisions, was too small, this instrument being the least sensitive, and therefore another cell was added to increase its deflection. Now, this increase in current increased the deflections on A and B also, and therefore shunts were put on them; but as it would never be possible to obtain a deflection as great as 90 degrees on these two instruments (their coils being parallel to the magnetic meridian) it may be asked, why should they be shunted at all? The reason is that if the needle of an instrument is deflected very nearly 90 degrees it requires the addition of very large resistances in the circuit to reduce this deflection down to a very small angle. For the purpose of calibration it is sufficient to deflect up to 70 or 80 degrees, and, therefore, the deflections on A and B were reduced to about 75 degrees by the addition of shunts of such values that only $\frac{1}{5}$ of the current passed through A and $\frac{3}{4}$ through B—that is, the multiplying powers of these shunts were 5 and $\frac{4}{3}$ respectively. There were then conveniently large deflections on each instrument, viz., $74\frac{1}{2}$ degrees on A, 74 degrees on B, and 187 divisions on C. This was with 94 ohms unplugged in the resistance box, which, together with the resistance of the instruments (A with shunt = 109 ohms, B with shunt = 144 ohms, and C unshunted = 253 ohms) made up an even 600 ohms; the main current was therefore $\frac{4 \text{ volts}}{600 \text{ ohms}}$ in amperes, or 6·6 milliamperes.

95. Calculations to be Made when Shunts are Employed.—From the above practical case of the use of shunts it will be

noticed that there are one or two small calculations to be made when they are employed, viz. :—

(1.) What is the ratio between the current in the main circuit and that passing through the instrument, shunted with a given resistance, or, in other words, what is the multiplying power of a given shunt?

If we call this ratio "n," it is clear that $\frac{1}{n}$th of the main current passes through the instrument, and therefore we must multiply the current passing through the instrument by n to

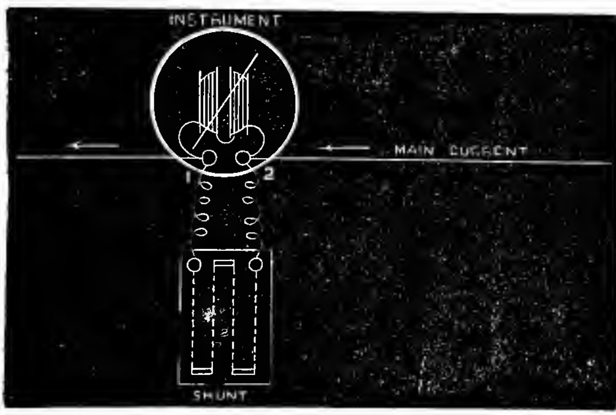


FIG 55.

find the value of the main current. Hence, the ratio n of the two currents is known as the "multiplying power" of the shunt.

In Fig. 55 an instrument is shown with a shunt attached to its terminals 1, 2; the main current arriving at terminal 2 divides between the two paths open to it, these two currents recombining again at terminal 1 and passing on. There must be a certain difference of potential between the terminals 1 and 2, or there would be no current, and this difference of potential is the same for both instrument and shunt, since both are joined

directly to the terminals 1 and 2. Therefore, by Ohm's law (para. 50) the currents through these two paths are proportional to their respective conductivities, and the sum of the two currents, *i.e.*, the main current, is also proportional to the combined conductivity of the two paths. From this consideration we have at once the relation between the main current and the current through the instrument expressed in the following proportion :—

$$\text{main current} : \text{current through instrument} :: \text{combined conductivity} : \text{conductivity of instrument.}$$

Representing, as above, by the letter n, the ratio between the two currents in the first and second terms of this proportion, we have

$$n = \frac{\text{combined conductivity}}{\text{conductivity of instrument}}.$$

It is, however, more convenient for practical use to express n in terms of resistances instead of conductivities, and we know that the conductivity of the instrument is

$$\frac{1}{\text{instrument resistance}},$$

and the combined conductivity of instrument and shunt is

$$\frac{1}{\text{instrument resistance}} + \frac{1}{\text{shunt resistance}};$$

whence we get

$$n = \frac{\dfrac{1}{\text{instrument resistance}} + \dfrac{1}{\text{shunt resistance}}}{\dfrac{1}{\text{instrument resistance}}}.$$

Dividing it out we have

$$n = 1 + \frac{\text{instrument resistance}}{\text{shunt resistance}},$$

which is the usual expression for calculating the multiplying power of a shunt. Expressed in words, *the multiplying power*

of a shunt is found by dividing the instrument resistance by the shunt resistance, and adding unity to the quotient.

(2.) When a given multiplying power, n, is required, what must be the resistance of the shunt? This is at once derived from the last result, from which we obtain

$$n-1=\frac{\text{instrument resistance}}{\text{shunt resistance}},$$

and therefore

$$\text{shunt resistance}=\frac{\text{instrument resistance}}{n-1}.$$

Applying this to the practical examples of the shunted instruments in para. 94, instrument A (of 544 ohms) was shunted with a shunt of multiplying power 5, and therefore the resistance of this shunt was

$$\frac{544}{5-1}=136 \text{ ohms.}$$

Instrument B (of 192 ohms) was shunted with a shunt of multiplying power $\frac{4}{3}$, or $1\frac{1}{3}$; the resistance of the shunt was therefore

$$\frac{192}{1\frac{1}{3}-1}=192\times 3=576 \text{ ohms.}$$

(3.) What is the combined or joint resistance of the instrument and shunt together?

The joint resistance of two or more resistances in multiple arc is the exact converse of their combined conductivity (para. 41). The combined conductivity as given before is

$$\frac{1}{\text{instrument resistance}}+\frac{1}{\text{shunt resistance}},$$

which, when brought to a common denominator, equals

$$\frac{\text{shunt resistance}+\text{instrument resistance}}{\text{instrument resistance}\times\text{shunt resistance}},$$

and the required joint resistance is the reciprocal of the above. We have, therefore,

$$\text{joint resistance}=\frac{\text{product of the two resistances}}{\text{their sum}}.$$

If, as is usually the case, the shunt is arranged for a definite multiplying power, n, we may arrive at the joint resistance from the value of n by a very quick and easy way. For it will be seen that if the expression above for the combined conductivity be split into two factors, one of which is the conductivity of the instrument, we have the combined conductivity

$$=\frac{1}{\text{instrument resistance}}\left(1+\frac{\text{instrument resistance}}{\text{shunt resistance}}\right),$$

and the remaining factor enclosed in the bracket will be recognised as n.

Therefore the

$$\text{combined conductivity} = \frac{n}{\text{instrument resistance}},$$

and the reciprocal of this gives the

$$\text{joint resistance} = \frac{\text{instrument resistance}}{n}.$$

This latter expression is very frequently used, since n is generally known first and the joint resistance calculated afterwards.

Applying this again to the practical examples in para. 94, the joint resistance of A and shunt was

$$\frac{544}{5}=109 \text{ ohms},$$

and that of B and shunt

$$\frac{192}{1\frac{1}{3}}=192\times\tfrac{3}{4}=144 \text{ ohms}.$$

Although the calculations with shunts have taken some time to describe in words intelligently, it will be seen that the calculations themselves are very simple.

96. Calculation of Actual Current through each Instrument and Plotting of Curves.—Continuing the details of the practical example of the three instruments in para. 94, after once adjusting the shunts, these have not to be altered again during

the whole test, and therefore there is in the circuit a permanent resistance, comprising

Joint resistance of A and shunt.........	109 ohms
,, ,, B ,, ...	144 ,,
Resistance of C alone	253 ,,
Battery	negligible resistance
	506 ohms

and with the permanent addition of 94 ohms in the resistance-box, the total resistance was made an even 600 ohms, this being increased by 100 or 1,000 ohms at a time, so that the resistance in circuit was conveniently always a multiple of ten. Two secondary cells of negligible resistance and 2 volts per cell having been used for this test, the main current passing through instrument C corresponding to each reading was

$$\frac{4 \text{ volts}}{\text{total resistance}} \text{ in amperes.}$$

The current through B corresponding to each reading was $\frac{3}{4}$ of the above main current, and, similarly, the current through A was $\frac{1}{5}$ of the main current.

The curves were then plotted for each instrument in precisely the same manner as explained previously, each curve showing the relation between the deflections and corresponding currents through the instrument.

97. The Tangent Galvanometer as a Standard Instrument of Comparison.—By a standard instrument is meant one in which the deflections of the needle are directly proportional to the currents passed through the coil, or one in which the current is proportional to some known function of the angle of deflection. This latter condition presents itself in the well-known *tangent galvanometer*, in which instrument the currents passing through the coil are proportional to the tangents of the angles of deflection of the needle. The reason for this action will be fully explained in the chapter on galvanometers, but it may be stated here that in order that an instrument may read tangentially the magnetic field of force produced by the

current must be uniform throughout the full range of movement of the needle. To ensure this the needle is made very short in length, not more than one-tenth the diameter of the coil. The coil itself is generally made circular in shape, though not necessarily so ; and the needle is pivoted or suspended horizontally at its centre. One of these instruments is shown in Fig. 56. The coil frame C C is circular, and the small needle hangs by a single silk

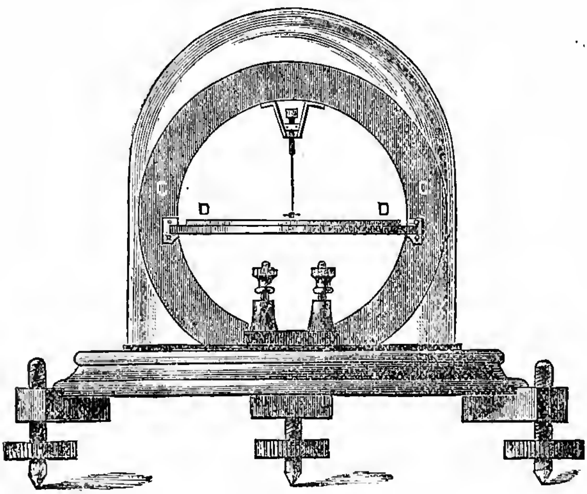


Fig. 56.

fibre in the centre of the coil, immediately above a graduated dial, D D. The needle is carried in a little stirrup of aluminium, to which a long, fine glass pointer is attached at right angles to the needle. The two ends of this pointer move over the scale divisions on D D, which are marked out in degrees of arc on one side of the coil, and in divisions proportional to the tangents of those degrees on the other side. Hence readings can be taken over either scale by reading from either end of the pointer, and if the tangent divisions are read the deflections are directly proportional to the strength of

current—that is to say, if the readings so taken and their corresponding currents were plotted together we should have a straight line, showing them to be absolutely proportional to each other. If, however, the *angles* of deflection were read off and plotted with the corresponding currents producing them, we should have exhibited a true tangent curve. It may be of interest to show this curve and the method of constructing it.

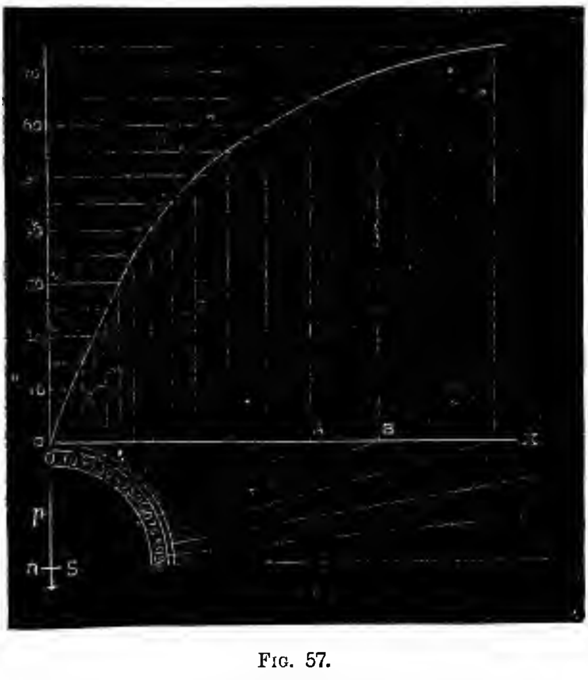


Fig. 57.

98. **Graphic Method of Constructing a Tangent Curve.**—In Fig. 57 the needle of a tangent galvanometer is shown at $n\,s$, with its pointer, p, attached to it at right angles. When there is no current passing through the instrument, the pointer will be at zero, and the needle will lie in the same plane as the coil (not shown). Now, when currents of varying strengths are

passed through the coil the needle will be deflected, carrying with it the pointer, which moves over a scale graduated in degrees, from which the angles of deflection can be read off. Now, the currents are not proportional to these angles of deflection, but to their tangents; that is, a current causing a deflection of, say, 60 degrees would not be twice the strength of current causing 30 degrees deflection, but the current strengths would be in the proportion of tan 60 to tan 30. Graphically, we may represent these tangents by distances along the line O X, by drawing lines from the centre of the needle to the various angles on the scale, and producing them till they cut the line O X; the distances from O are then proportional to the tangents of the corresponding angles. For example, taking the two angles 65 degrees and 70 degrees, the respective distances O A and O B are proportional to the tangents of these angles, and so on for every other angle. For, calling the distance from O to the centre of the needle the radius of the arc, we have the actual tangents of the angles moved through by the pointer *p* (and therefore by the needle, since it is fixed to the pointer), equal to their respective distances along O X divided by the radius; thus,

$$\tan 65^\circ = \frac{O\,A}{\text{radius}},$$

and

$$\tan 70^\circ = \frac{O\,B}{\text{radius}},$$

and so on for all angles. Now, since the radius is common to all, it is evident that distances along O X simply are proportional to the tangents of the angles.

Hence the points on the line O X, found by lines drawn from the centre of the needle through the angles to the horizontal line O X indicate by their distance from the point O the magnitude of the tangents of those respective angles. But the currents are also proportional to the tangents of the angles; therefore the distances from O along O X are proportional to the currents which would cause the corresponding angles of deflection, and we may therefore regard the line O X as

marked off in strengths of current, proportional to the currents required to produce those angles of deflection. Now, if we mark off a vertical scale of degrees from the point O, the points on the tangent curve may be found in the following manner:—Lines drawn vertically from the current divisions on the horizontal ordinate must each intersect a horizontal line drawn from its respective angle on the vertical ordinate, and the points of intersection so obtained are actual points on the tangent curve. Thus, where the perpendicular from A cuts the horizontal from 65 degrees, the point of intersection is one point on the tangent curve. Now, on joining together all these points of intersection, the tangent curve is obtained as shown in the figure. This is, of course, precisely the same thing as a calibration curve of any tangent galvanometer, and it will be of interest to compare the shape of this curve with the preceding actual curves which have been plotted for various indicators not constructed tangentially. For instance, the curve in Fig. 52 is of the same general shape as the tangent curve, while the others in Figs. 48 and 53 are of a different type. A curve may, however, greatly resemble, and yet not accurately be, a tangent curve. It will be noticed that the tangent curve is straight along its first portion from O to about 25 degrees, which means that the angular deflections on a tangent galvanometer are directly proportional to the currents producing them up as far as 25 degrees, beyond which the currents are proportional to the tangents of the angles. And it will also be noticed that as the currents are increased the increase of angular deflection becomes less and less, and that a deflection of 90 degrees could never be produced, since this would require an infinite strength of current. Graphically this last fact would be evident by continuing the drawing of the curve in the same way as shown above, when it would be seen that the curve could never reach the height corresponding to 90 degrees, however far it was produced.

99. Calibration of an Indicator by Comparison with a Tangent Galvanometer of similar Sensitiveness.—The current

indicator to be calibrated is connected in series to the tangent instrument, an adjustable resistance box, and a few cells. Now, since we have a standard instrument in circuit from whose indications we know at once the ratios of the strengths of current flowing, it is not necessary to know the E.M.F. of the battery used, nor to have any idea of its internal resistance; nor do we require to know the values of the resistances added to the circuit, as in the previous method (para. 87). The readings for the curve will then be taken on each instrument for every change in the current, starting with no resistance and adjusting the number of cells to give conveniently large deflections of about 80 degrees on each instrument, and then increasing the resistance step by step, noting the angular deflections on both instruments corresponding to each change.

100. The "Calibration Constant" of a Tangent Galvanometer.—The actual values of the currents on the tangent galvanometer are readily found when the deflection corresponding to one particular current is known, all the others following proportionally from that. For example, the tangent instrument in Fig. 56 requires a current of 2·43 milliamperes to deflect the needle 45 degrees. From this determination, which is easily made by passing a known current through the instrument, we may find the current C corresponding to any other deflection, say 60 degrees, by simple proportion :—

$$C : 2{\cdot}43 :: \tan 60 : \tan 45,$$

and since tan 45 is equal to unity

$$C = \tan 60 \times 2{\cdot}43 = 4{\cdot}21 \text{ milliamperes.}$$

Hence the tangent of any angle of deflection on this galvanometer, multiplied by 2·43, gives the current in milliamperes corresponding to that particular angle of deflection, and the number 2·43 is known as the "*calibration constant*" for this particular tangent instrument. Hence *the calibration constant of a tangent galvanometer is the number which, when multiplied by the tangent of the angle of deflection, gives the current passing, and this*

number is equal to the current which causes the needle to deflect to 45 degrees. The values of the tangents of the various angles are found by reference to a table of tangents (*see* end of this book).

The calibration curve of the indicator may now be plotted in the usual manner, by marking off its angular deflections on the vertical ordinate, and the corresponding currents, determined on the tangent galvanometer as above, on the horizontal ordinate.

If the curve is taken with the object simply of having a record of the relation between currents and deflections, and it is not desired to have the actual values of the currents marked on the ordinate, all that is necessary is to mark out on the horizontal ordinate the tangents of the deflections on the

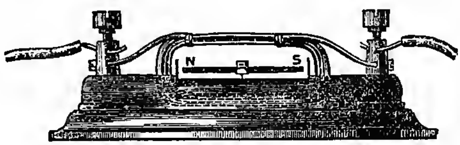


FIG. 58.

tangent instrument and the corresponding angles of deflection on the indicator on the vertical ordinate. It should be borne in mind, however, that the curve is the same whether the actual values of the currents are determined or not. But it is such an evident advantage to have the actual current values, and so little extra trouble to determine them, that the methods for calibration which are here given include their determination.

101. Practical Example of Calibration by Comparison.— In Fig. 58 is shown a current indicator for indicating currents of greater strength than could be measured with the instruments previously considered. The needle is shown at $n\ s$, and is pivoted on a point fixed in the centre of an ordinary compass box marked round in degrees. The coil is of rectangular shape, and consists of a few turns

of No. 18 insulated copper wire, the lower half of the coil being let into the wood base. The curve determined for this instrument is shown in Fig. 59. Now, since the currents reach as high as one ampere and over, the cell used for the test and the tangent galvanometer used as an instrument of comparison must both be of low resistance, otherwise we shall not get sufficient current. For example, with one cell of two volts E.M.F. giving one ampere of current the total resistance of instruments and cell could not exceed two ohms. Hence, the tangent galvanometer used was one with a thick wire coil of a few turns and very low resistance, the cell used was a secondary cell of about $\frac{1}{50}$th of an ohm, and an adjustable resistance frame containing spirals of No. 18 German silver wire, by which the extra resistance in the circuit could be adjusted from one to twelve ohms, was also included in the circuit. As mentioned above, the actual resistances in the circuit are not taken into account, since the tangent instrument gives the values of the currents. The readings taken were as follows:—

Angles of deflection on indicator.	Currents on tangent galvanometer.
82	1·69 amperes
78	1·3 ,,
76	1·05 ,,
74	·92 ,,
69	·67 ,,
58	·39 ,,
37·5	·14 ,,

From these observations the curve in Fig. 59 is plotted, and as it presented a resemblance to a tangent curve in general form, the writer has worked out a few values for the true tangent curve shown by the dotted lines, by which the wide difference which would be anticipated between this instrument and a true tangent can be seen. To do this, one point on the curve must be taken as a standard. Taking for this the first point on the upward slope of the curve, viz., 37·5 degrees with ·14 ampere we may proceed to find the values of the

currents for the other deflections, supposing the instrument for the time being to be a tangential one. Finding the current C for the next deflection of 58 degrees, we have, by simple proportion—

$$C : \cdot 14 :: \tan 58 : \tan 37 \cdot 5.$$

Hence,
$$C = \cdot 14 \times \frac{\tan 58}{\tan 37 \cdot 5},$$

$$= \cdot 14 \times \frac{1 \cdot 6}{\cdot 767} = \cdot 29 \text{ ampere.}$$

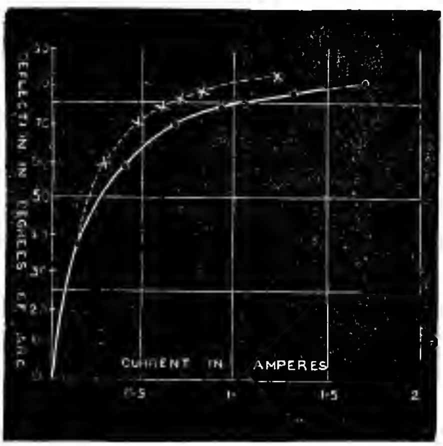


Fig. 59.

Now, 58 degrees was the deflection actually produced by ·39 ampere, but if the instrument had followed the tangent law the same deflection would have been produced by ·29 ampere. This is due to the large dimensions of the needle in comparison to the size of the coil. As the needle is deflected its poles move into a weaker field owing to its length, and hence a stronger current is necessary to produce a given deflection than would be the case if the needle was of sufficiently short length

to prevent its moving out of the uniform field during any part of its movement. Working out a few more points similarly to the above, keeping the first point, viz., that for the weakest current, as the standard, and plotting them on the same curve paper, the difference between the indicator in question and a true tangent instrument is made evident.

CHAPTER IX.

Magnetic Fields and their Measurement.

Section I.—*Permanent Magnet Fields.*

102. Preliminary.—One of the first considerations towards the understanding of the action of galvanometers is a clear conception of the delineation of the magnetic field set up in and around a coil of wire through which a current is flowing. The needle of a galvanometer is retained in the magnetic meridian, when there is no current through the coil, by the horizontal component of the magnetic field of the earth, and when it is deflected away from its position of rest we must take into account both the magnetic field due to the current which causes that deflection and the field of the earth tending to keep it in the meridian.

The relation between the magnetic field produced at the centre of a coil, the number of turns of wire composing the coil, the strength of current, and the angle of deflection of a magnetic needle free to move in the centre of the coil, is intimately connected with the principle of galvanometers, and it will be shown that the strength of a current may be measured in absolute units by means of a tangent galvanometer when the number of turns of wire, the mean radius of the coil, the horizontal magnetic field of the earth, and the deflection produced by the current are known.

To understand fully these points there should first be a clear conception of the fundamental units employed in the measurement of the intensity of a magnetic field, current, and strength of magnetic pole. To explain these and the manner in which they are co-related we shall discuss the centimetre-gramme-

second system, from which the units are derived, and deal in the following order with the magnetic fields exhibited by—

1. Permanent magnets.
2. Electro-magnets.
3. Coils or solenoids.

103. Magnetic Field of Permanent Magnet.—There is nothing visible nor are there conditions in any way capable of affecting our senses in the immediate neighbourhood of a magnetised steel bar, yet we are well aware that pieces of iron or steel,

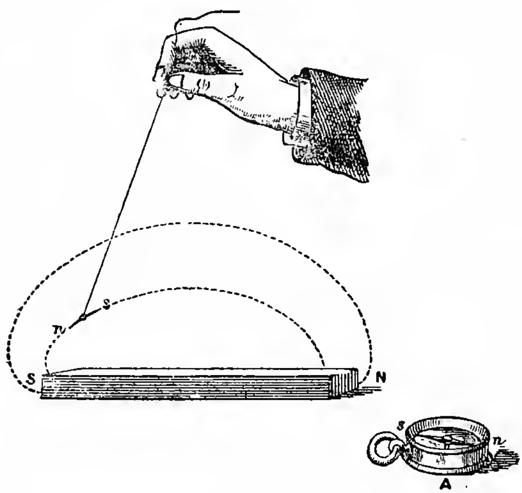


FIG. 60.

whether previously magnetised or not, are powerfully acted upon by some invisible force when they are brought into the space surrounding a bar magnet. The whole of this space, which is capable of exercising magnetic force on magnetised bodies, is termed a "magnetic field" of force.

If a small compass needle, *n s* (Fig. 60), be attached at its centre to a thread of silk, so that it is balanced in a horizontal position when freely suspended by the silk, it may be employed

to explore the character of the field of force round a bar magnet, N S, as shown. A good deal may be learnt by doing this simple experiment. The *direction* of the force will be the direction in which the needle sets itself. Of course, there is the movement here of "translation" or bodily movement of the needle, as well as its turning about its own centre, but by moving the hand about, so altering the distance of the needle from the bar and its position with regard to the ends of the bar, it will be found that when the needle is at rest in any position it will always seem to lie in the direction of curves existing between the two ends of the bar, as shown by the

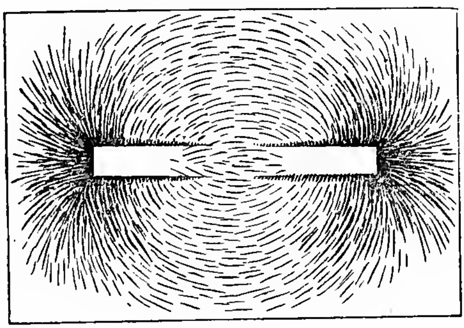


Fig. 61.

dotted lines (Fig. 60). It will also appear that the compass needle is much more powerfully attracted in some parts of the "field" than in others. This can be told by the rapidity of the oscillations it makes before coming to rest.

Close to the ends of the bar, or at the "poles," the strongest force will be observed. In this country we term the north pole of a magnetic needle that pole which turns towards the geographical north, and a mark or cut is made on the north end of most magnets to distinguish this pole. The poles of the bar magnet and compass needle being known, it will be noticed that the latter invariably set themselves in such a direction that poles next to each other are opposite in name. By covering the bar with a sheet of stiff cardboard, or a plate of glass,

and letting finely divided iron filings fall over it, at the same time gently tapping the plate, the general character of the "field" is observed. Fig. 61 is a drawing from an actual field so taken, and exhibits the direction of the curves of force in the surrounding space. This, of course, only shows the field in one plane, but we should have the same character of field whichever side of the bar was uppermost; and hence the whole of the space immediately surrounding the bar is occupied by these curves of force.

Similarly Figs. 62 and 63 are drawings of magnetic fields taken on a powerful compound horse-shoe steel magnet.

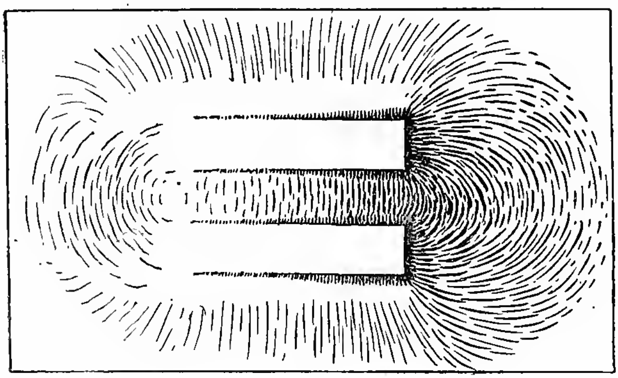


Fig. 62.

104. Lines of Force.—Faraday conceived that the space in the immediate vicinity of magnets was occupied by "lines of force," existing precisely in the directions assumed by the iron filings when sprinkled over the field, and this idea lends itself very conveniently to the explanation of the various observed effects. It is also convenient to assume a direction for the lines of force; for suppose that the bar magnet in Fig. 60 was turned round, putting the north pole to the left and the south to the right, the effect on the two needles would be to turn them to the right-about-face, showing that although the lines of force are still there, something has happened analogous

to changing their direction. As a basis of reasoning it is assumed that the direction of the lines of force is that in which a free

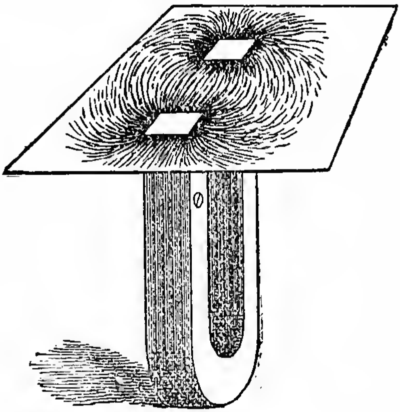


FIG. 63.

north pole would be moved along the field. As a north pole cannot exist in a "free" state without its corresponding south

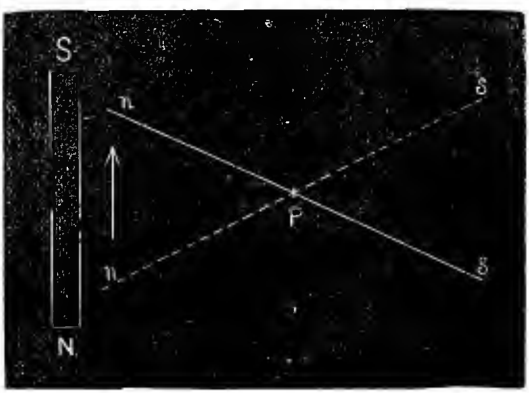


FIG. 64.

pole, the nearest practical way of showing this is by taking a long magnetised steel wire, *n s* (Fig. 64), pivoted at P, and

holding it near to the bar magnet N S in the dotted line. The action of the distant south pole of the steel wire we may neglect, as it is comparatively so far away, and we have the north pole *n* separately subjected to the action of the bar magnet; and if the long magnet is allowed free movement its north pole will move in the direction of the arrow from N to S, being repelled by one pole and attracted by the other. The assumed direction of the lines of force is therefore from the north pole of a magnet towards its south pole through the surrounding space, but from the south to the north through the interior of the magnet, because all lines are supposed to form a closed circuit, which is completed through the mass of the iron of the magnet. It is also assumed that the greater the intensity of a magnetic field the greater the number of lines of force pass through a given area.

105. **Permanent Magnet Controlling Field.**—A galvanometer is usually provided with a permanent magnet, by means of which its sensitiveness may be reduced to any desired extent. This is placed vertically above the needle, and can be shifted vertically so as to vary its effect upon the latter. If it is lowered closer to the needle its force on the latter is increased, and a stronger current through the coil of the instrument will be required to deflect the needle through a given angle—in other words, the sensitiveness of the galvanometer has been reduced. When the current is stopped, the needle is controlled or directed back to the plane of the coil by the directing action of the field of the permanent magnet. Hence, the magnet is known as a *controlling* or *directing magnet*, and the magnetic field produced by it in the region of the needle's movement a *controlling* or *directing field.* This controlling magnet is generally placed in such a position that its magnetic field is in the same direction as that of the earth's horizontal magnetic field. This is not absolutely necessary in ordinary tangent galvanometers, as the tangent law holds good whether the controlling field is that of the earth or a controlling magnet, or both combined; that is, the instrument follows the tangent law equally as well when the controlling magnet keeps

the needle parallel to the plane of the coil when no current is on, even though the plane of the coil is not in the magnetic meridian. When this is the case the actual controlling force on the needle is the resultant of that due to the earth and that due to the permanent magnet. In instruments, however, reading absolutely in terms of the controlling field, such as Sir William Thomson's graded potential and current galvanometers, it is absolutely necessary, if a controlling magnet is used, to place it so that its field is in the same direction as that of the earth. This is easily done by first levelling and moving the

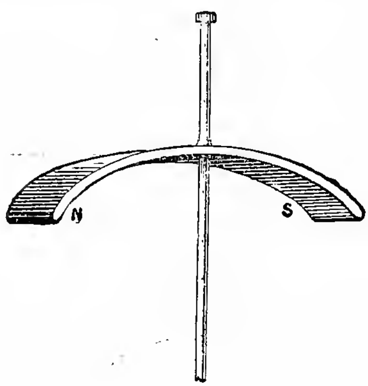


Fig. 65.

instrument till the needle is in the same plane as the coil (shown by the pointer being at zero) and then placing the magnet over the needle and varying its position till the pointer stands at zero again; the two fields are then parallel. In Fig. 65 is shown the controlling magnet of a Thomson reflecting galvanometer. A split brass tube attached to the magnet causes it to grip the vertical guide rod, and remain in any position in which it is placed. The magnetic field surrounding this magnet may be understood by the iron-filing diagram in Fig. 66, which was taken with this magnet. Now, in order that this magnetic

field may act on the needle in the same direction as that of the earth, the south pole of the magnet must point towards

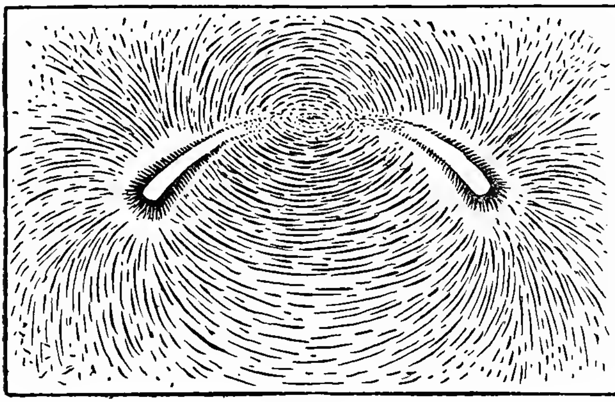


Fig. 66.

the north pole of the earth. For if the earth's field is represented by the straight arrows N S in Fig. 67, where

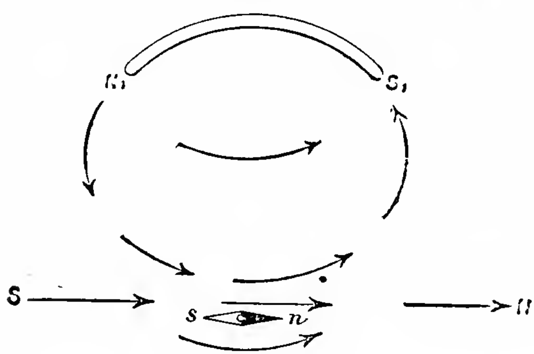


Fig. 67.

these letters are due north and south respectively, and the curved arrows represent the direction of field due to the mag-

net $N_1 S_1$, it is evident the two fields will agree in direction when the magnet is so placed—in other words, the lines of force of the two fields must be in the same direction for them to act together; and the *north* pole of a magnetic needle, *n s*, free to turn on its centre, always points *towards* the direction in which the lines of force are assumed to act (para. 104). It is well to bear this last fact in mind.

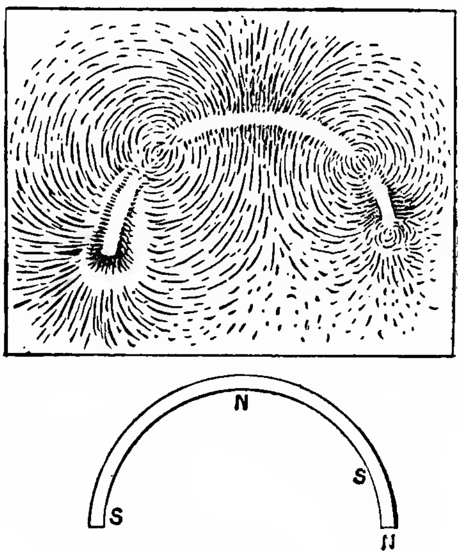


FIG. 68.

An important point is that the controlling magnet should be magnetised regularly—that is, as in Figs. 65 and 66, where only two poles appear. If more than two poles exist in the magnet we get an irregular field, as shown in Fig. 68, which is taken from an actual magnet with two distinct additional poles. Such additional poles are known as "*consequent poles.*" The diagram of the field of this magnet, taken with iron filings, shows very well the three complete magnetic circuits

of lines of force which exist in it. Such a state of affairs had probably been due to this magnet being placed near other magnets, or touching them at different points. To magnetise a controlling magnet regularly the best way is to wrap it round with a layer or two of insulated wire, and then pass a strong current through the wire, tapping the magnet with a mallet to reduce the friction between the molecules of the iron, and

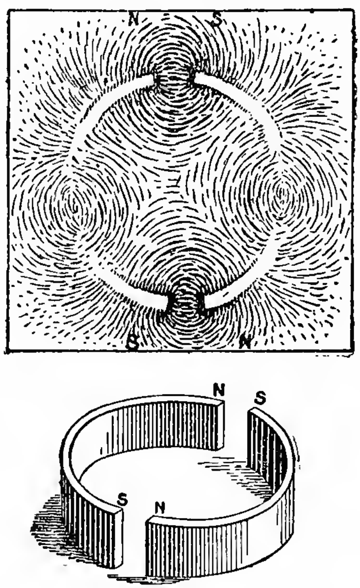


Fig. 69.

allow them to take a permanent set in the direction of magnetisation. A perfectly symmetrical magnetic field is exhibited in Fig. 69, where two semi-circular controlling magnets of an ordinary mirror-signalling instrument are placed almost together, with opposite poles *vis-à-vis*. Each magnet completes its own magnetic circuit through the other—that is, instead of all the lines of force branching out from one pole and bending towards the opposite pole of the same magnet, as in the single

magnet in Fig. 66, the greater part now branch towards the opposite pole of the other magnet and complete their circuit through its mass, this being an easier path for them than through the air.

106. The Earth's Magnetic Field.—If a light strip of steel be mounted at its centre on a horizontal axis so that it is perfectly balanced and free to move in a vertical plane, and it is then taken off the pivot and subjected to the process of magnetisation, it will be found on replacing it in its bearings that it will appear to have one end heavier than the other; that is, the rod will take up a definite inclined position, and will not remain in any other position but this. This effect is due to the magnetic field of the earth, and in our latitude it is the north pole of the pivoted magnet which is so "inclined" or "dipped" downwards. By the exploration of the magnetic field of an ordinary bar magnet (as shown in Fig. 60) it may be noticed that the suspended needle "dips" when near either pole; but when over the centre or equator of the magnet it remains horizontal, and we account for the effects observed on the surface of our globe by imagining its axis to be a huge magnet, whose north pole lies towards the geographical south, and whose south pole lies towards the north. If we shift the position of the magnetised steel strip until it is free to move in a vertical plane *coinciding* with the direction of the magnetic meridian, we have the greatest angle of dip—that is, the magnetised strip takes up the direction in which the "intensity" of the earth's field is greatest.

The angle which the strip or needle so inclined makes with the horizontal is termed the angle of "*Dip*" or "*Inclination.*" As we go further north the "angle of dip" increases, and we should expect to find the needle nearly vertical in regions near the geographical north pole. Similarly, on going south the "dip" decreases until there is no dip at all. We are then on the "magnetic equator" of our globe, and this equator, in which there is no angle of dip, runs very nearly through Aden, Madras, Penang, and Borneo, being several degrees north of the geographical equator. Going further south still

the needle "dips" the other way—that is, with its south pole downwards, the angle of dip at the City of Melbourne being about the same as in London now—viz., about 67 degrees. At the time it was first observed in the year 1576, it was 71° 50′ in London, and it steadily increased, until in 147 years after that time it had reached 74° 42′, since which, down to our present time, it has been slowly decreasing.

It is not within the scope of these Papers to describe the methods of taking measurements of terrestrial magnetism; these are continually carried out at the different Observatories, with instruments of great precision, as for example at Kew and Greenwich, and continuous records are kept.

The "*Total Intensity*" of the earth's field is the intensity measured in a direction inclined to the horizontal by the angle of dip or inclination, and in the vertical plane of the magnetic meridian. In order to express this total intensity in units of magnetic measurement, or indeed to express the strength of any magnetic field in units, we must first define the unit magnetic pole and the unit of force.

107. The Unit of Force on the centimetre-gramme-second system is that force which, acting on a mass of one gramme, free to move, imparts to it every second an acceleration or increase of velocity of one centimetre per second, or, in other words, that force which, acting for one second on a gramme, gives it a velocity of one centimetre per second. This unit of force is termed the "*Dyne*," and is equal to about the weight of 1·02 milligramme. If a mass of one gramme were allowed to fall by the action of gravity, the force exerted on it would be 981 dynes, because its acceleration would be 981 centimetres per second every second. Now, all bodies fall at the same rate, from the same height, in vacuo, whatever their mass, since one gramme experiences a force of 981 dynes, two grammes double that force, and so on, the ratio between force and mass being constant; and this ratio is the acceleration due to gravity. Hence the acceleration due to gravity is constant and equal to 981 centimetres per second per second, and the force in dynes exerted on any body in the

earth's gravitation field of force is found by multiplying this acceleration by the mass of the body in grammes. As an example of this, take one kilogramme (1,000 grammes): the force of gravitation on each gramme is 981 dynes, and, therefore, on one kilogramme is 981,000 dynes, and it is precisely this force acting on the mass which gives it what we call weight. Again, one English pound is equal to 453·6 grammes, and therefore its weight is equivalent to $453{\cdot}6 \times 981 = 44{,}500$ dynes nearly.

108. The Unit Magnetic Pole.—We can now define the unit magnetic pole, which on the C.-G.-S. system is that pole which, placed at one centimetre distance from an exactly similar pole, exerts a force upon it of one dyne. Here we have a measurable quantity, and having established the unit pole, we can measure the strength of any pole in C.-G.-S. units by the force it exerts on a unit pole placed at one centimetre distance from it. Coulomb established by means of his torsion balance, described in most text-books, the law of inverse squares with reference to magnets—that is, that the force exerted between two magnetic poles varied directly as the product of their individual strengths, and inversely as the square of the distance between them. If two magnetic poles, when placed five centimetres apart, experienced a force of four dynes between them, we should know that the product of the strengths of the two poles was equal to $(5)^2 \times 4 = 100$, and if the strength of one of the poles was equal to unity, the other pole would be 100 units in C.-G.-S. measure. In that case the unit pole would be acted upon by a force of four dynes, due to a magnetic pole of strength of 100 units placed five centimetres away. In other words, the unit pole would be in a magnetic field equal to four C.-G.-S. units; this brings us to unit magnetic field.

109. Magnetic Field of Unit Intensity.—In the C.-G.-S. system the unit magnetic field is that field which will exert a force of one dyne on a unit magnetic pole placed in it, so that when we speak of a magnetic field of so many C.-G.-S. units it means that a unit magnetic pole placed in it would experience

a force equal to that number of dynes. Sometimes a field is expressed as equal to so many dynes; this is concise, but must always be taken to mean that the field would exert so many dynes force on a unit pole placed in it. The force in dynes exerted on a magnetic pole when placed in a magnetic field is equal to the strength of the field in C.-G.-S. units multiplied by the strength of pole. Hence a magnetic field is numerically equal to a force divided by pole strength, and can, therefore, only be expressed as a force when the pole is of unit strength. There is no name given to the unit magnetic field, so that the strength of a field is simply expressed as equal to so many C.-G.-S. units. It is also assumed that the number of lines of force per unit area (the area being at right angles to the direction of the lines) is a measure of the intensity of a magnetic field, and the unit intensity of field is taken as one

FIG. 69A.

line of force per square centimetre area. This assumption is a very convenient one for working out problems in magnetic fields.

Let a bar magnet (Fig. 69A) be laid horizontally in the magnetic meridian, with its north or marked pole pointing northwards, and its south pole due south. The lines of force due to the magnet and the earth are then in the same direction, in a line through the axis of the magnet, and the actual strength of field at any given point p on this line will be the sum of the two above fields at that point. That due to the earth is the horizontal component of the earth's magnetic field, and is denoted by the letter H. Its value is ·18 C.-G.-S. units in London—that is, unit pole (para. 108) experiences ·18 dyne force acting upon it horizontally in London. Suppose the above bar magnet (Fig. 69A) to be 11·06 centimetres long, and each pole to be 82 C.-G.-S. units in strength, the point p

being 8·94 centimetres distant from the S pole. We have, then, at the point *p* *unit* magnetic field, for the force on a unit north pole placed at *p* would be (para. 108)

$$\frac{82}{(8{\cdot}94)^2} = \frac{82}{80} \text{ dynes (of attraction) due to pole S,}$$

and $\dfrac{82}{(8{\cdot}94 + 11{\cdot}06)^2} = \dfrac{82}{400}$ dynes (of repulsion) due to pole N,

the *net* action due to the magnet being, therefore,

$$\frac{82}{80} - \frac{82}{400} \text{ dynes (of attraction),}$$

FIG. 69B.

which is equal to

$$\cdot 82\ (1\tfrac{1}{4} - \tfrac{1}{4}) = \cdot 82 \text{ dyne.}$$

But the earth's field H is acting on the unit pole at *p* in the *same* direction with a force of ·18 dyne; therefore the total action on the unit pole is equal to

$$\cdot 82 + \cdot 18 = 1 \text{ dyne (of attraction).}$$

In other words, we have *unit* magnetic field at *p*, and a similar reasoning would show that there is precisely the same unit field at *p'*, a point at the same distance from the N pole.

If, now, the bar magnet were turned to the rightabout, but still kept in the meridian (Fig. 69B), the field of the earth would act in opposition to that of the magnet. We should now find that *unit* magnetic field would exist at a point *p*, distant from the N pole by 7·58 centimetres (this unit field being

opposite in direction to the unit field in the last example). For the force on a unit north pole at p would be (para. 108)

$$\frac{82}{(7{\cdot}58)^2}=\frac{82}{57{\cdot}45} \text{ dynes (of repulsion) due to pole N,}$$

and $$\frac{82}{(7{\cdot}58+11{\cdot}06)^2}=\frac{82}{347{\cdot}45} \text{ dynes (of attraction) due to pole S,}$$

the *net* action of magnet being, therefore,

$$\frac{82}{57{\cdot}45}-\frac{82}{347{\cdot}45}=1{\cdot}18 \text{ dyne (of repulsion).}$$

Now the earth's field exerts an attractive force on the unit north pole p equal to ·18 dyne. This must, therefore, be subtracted from the force due to the magnet to obtain the actual force on the pole at p, which is then

$$1{\cdot}18-{\cdot}18=1 \text{ dyne (of repulsion),}$$

that is, we have unit magnetic field at the points p and p'.

It will now be understood what is meant by saying that the total intensity of the earth's magnetic field is equal to ·47 C.-G.-S. units. The total intensity of the earth's field does not come into the calculations with galvanometers, since the needles are not used pivoted in the angle of dip; all needles moving in a horizontal plane being acted upon or controlled by the earth's horizontal field H. It is evident that, knowing the value of the total force of the earth in the direction of the dipping needle, and the angle of inclination which this force makes with the horizontal, we can resolve it into two component forces, one acting horizontally, the other vertically

Resolving it, we have the

horizontal component = total intensity × cosine of angle of dip.

Thus in the year 1879 the total intensity in London was ·4736 C.-G.-S. units, and the angle of dip was 67° 42′. From this we get the horizontal component

$$= {\cdot}4736 \times \text{cosine } 67^\circ\ 42'$$
$$= {\cdot}4736 \times {\cdot}37945 = {\cdot}1797 \text{ C.-G.-S. units.}$$

Similarly resolving for the vertical component we have the

vertical component = total intensity × sine of angle of dip.

Therefore for the same year the vertical component was

$$= \cdot 4736 \times \sin 67^\circ\ 42'$$
$$= \cdot 4736 \times \cdot 9252 = \cdot 4381 \text{ C.-G.-S. units.}$$

The value of the horizontal magnetic field of the earth is on the increase, and is now equal to ·18 unit in London, while the total intensity is decreasing gradually in value.

110. The Magnetic Meridian.—At any given place on the earth's surface the magnetic meridian is the vertical plane (through that place) in which the total intensity of the earth's magnetic field is a maximum. Of the two component intensities, the horizontal and the vertical, into which, as before shown, the earth's field may be resolved, the former is necessarily a maximum in the same vertical plane as the total intensity, viz., in the magnetic meridian; hence the direction of this meridian may be seen by the direction in which a horizontally-pivoted magnetic needle sets itself, while the latter component, viz., the vertical, is the same in intensity in all vertical planes at one place. If, for example, we take a vertically-pivoted magnetic needle, whose horizontal axis of motion is through its centre of gravity, and move it round bodily, so that the vertical plane in which the needle moves is gradually altered until a vertical plane is found, in which the needle comes to rest in an absolutely vertical position, we know that the plane so found is at right angles to that of the magnetic meridian, because it is only in this plane that the horizontal intensity could be *nil*, and the vertical alone act. Having found this plane in the foregoing manner, we know that the magnetic meridian is in a plane 90 degrees removed from it. This method of determining the plane of the magnetic meridian is employed at Kew Observatory, in order to place the "dipping needle" correctly in that plane before taking readings of the angle of "dip" or "inclination." Full descriptions of the instruments employed, and the methods of

measuring the horizontal intensity, and the angles of inclination and declination, with the corrections to be taken into account, will be found in Gordon's "Physical Treatise on Electricity and Magnetism," Vol. I., and in Stewart and Gee's "Elementary Practical Physics," Vol. II. The plane of the magnetic meridian does not always coincide with that of the geographical meridian as determined astronomically. From data accumulated since 1576 it is found that in London the magnetic and geographical meridians coincide about every 320 years. Since their coincidence 231 years ago the magnetic meridian became more and more west of the true meridian, until, after a period of 160 years, a maximum was reached, and it then commenced to return. The angle between the two meridians is called the angle of *declination.* The latest determinations of the magnetic elements for the completion of the year 1887 have been kindly supplied to the writer by the Superintendent of the Kew Observatory, and are as follows:

Mean Declination, 18° 7′ west;
Mean Inclination, 67° 37′;
Mean Horizontal Intensity, ·1810 C.-G.-S. unit.

It will also be understood that the intensity of the horizontal magnetic field varies according to the latitude; for instance, on the magnetic equator, or line of no dip, the horizontal intensity is equal to the total intensity, and the further north or south we go the weaker does the horizontal intensity become, the difference in its value being noticeable even in Great Britain, where, for instance, its value in the town of Inverness is ·15 C.-G.-S. units, while in London it is ·18, the difference of latitude being about 6 degrees.

111. Uniform Magnetic Field.—A magnetic field is said to be "*uniform*" when the force exerted on a magnetic pole of given strength is the same at all points in that field. Throughout the space described by the movement of any given magnetic needle suspended or pivoted horizontally, as in galvanometers and indicators, we may regard the earth's

horizontal field H as absolutely constant in value; in other words, such a needle is suspended in a "uniform" field, and the force exerted on its two poles would be the same for all positions it could possibly take up. The number of lines of force per square centimetre would also, by definition, be constant at all points in a uniform field, and therefore the lines of force would necessarily be parallel lines.

112. Comparison of the Intensities of Magnetic Fields.—The relative intensities of two or more magnetic fields, or the relative strengths of field at two or more different points in a magnetic field which is not uniform, may be determined by the method of vibrations. A small magnetic needle, horizontally suspended or pivoted, is placed successively at the points

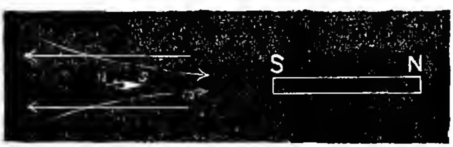


FIG. 70.

where the magnetic fields are to be compared, and after having been set oscillating, the number of oscillations made in a given time are counted. The cause of the oscillations is evidently the magnetic force (equal to the product of the strength of one pole of the needle and the strength of the field) acting at both ends of the needle, and tending to bring it to rest in a line parallel to the lines of force of the field. If there were no force there would be no oscillation. We can observe this by annihilating the force acting on the needle.

An ordinary compass needle, if turned round out of the magnetic meridian and then allowed to swing, will continue to oscillate about the meridian till it finally comes to rest in it. The force causing these oscillations can be weakened or annihilated by destroying *either* of the two factors composing it. That is, we may do away with the force by destroying either

the magnetic field or the strength of the needle poles. If the needle ns (Fig. 70) is lying in the magnetic meridian, the earth's field H (shown by the straight arrows) may be neutralised by approaching a magnet, N S, which sets up in the neighbourhood lines of force (shown by the curved arrows) opposite in direction to those of the earth. As the magnet is approached, if the needle is turned round its centre and then allowed to oscillate back, it will be found that the oscillations become more and more sluggish, and a point will at length be reached when the needle will not oscillate at all, but remain in any position it is turned to. This principle of weakening the earth's directive force on a needle by means of a magnet placed, as shown, in the neighbourhood is employed sometimes with galvanometers, where it is required to render the instrument more sensitive, and increase the amplitude of the deflections with weak currents.

Or, again, we may destroy the force by demagnetising the needle—that is, doing away with the other factor on which the force depends. If the needle is demagnetised completely, as may be done by placing it within a helix, or coil of insulated wire, through which an alternating current is passing, or what is the same thing, if, instead of the magnetised needle, we substitute a piece of hard steel unmagnetised, we find that it will remain still in any position in which it may be placed; in other words, there is no force acting on it, and therefore the needle does not oscillate or take up a definite position. We have a mechanical example of this when a body is suspended vertically at the end of a length of fine thread. The top end of the thread being fastened to some fixed point, if the body is raised a little with the thread taut and then let go we have a pendulum action, and oscillations take place till the pendulum finally comes to rest in the vertical line of the earth's gravitation field of force.

In a similar manner to the magnetic needle above, we have here a force acting on the body, causing its oscillations, and composed of two factors, viz., the mass of the body and the acceleration of gravity. If we could destroy either factor, oscillation would cease, in consequence of the absence of any

force. The mass of the body could only be dispensed with by removing the body altogether from the thread, which would then do away with the force and the resulting oscillations. We could not, however, neutralise the effect of the acceleration of gravity by a counter field of force in the same manner as a magnetic field.

Incidentally, here, attention may be drawn to the analogy between a mechanical and a magnetic field of force. Mass in the one is analogous to strength of pole in the other, and acceleration in the one (force divided by mass) is analogous to intensity of field in the other (force divided by pole strength). If there could be such a thing as a separate pole it would move along a line of force in a uniform magnetic field at a constantly increasing velocity or acceleration.

We now come to the well-established law of compound pendulums, viz., that the value of a force causing such oscillations (when these are of small angle) is proportional to the square of the number of complete oscillations in a given time, or inversely proportional to the square of the time during which a given number of complete oscillations are made. The time of a complete oscillation or "period" is taken from the instant the needle passes a given point to the instant it reaches the same point again, moving in the same direction.

Suppose the intensities at two points, A and B, in a magnetic field are to be compared. If at A a magnetic needle makes, say, 60 oscillations per minute, and at B the same needle makes 20 per minute, the fields at those points are respectively in the proportion of $60^2 : 20^2$—that is, as 9 to 1.

113. Comparison of the Strengths of Poles of Magnets.—By the same method we may compare the strength of pole of two or more magnets. Let a short magnetic needle vibrate freely over a flat surface till it comes to rest in the magnetic meridian, then draw a straight line on the flat surface in the direction of the needle's length. This line will be the direction of the magnetic meridian. Now turn the needle round its centre away from the meridian, and let it oscillate. Count the number of oscillations, say 4 in 12 seconds—that is, 20 per minute in

the earth's field alone. Now, suppose we have two bar magnets, A and B, of equal length, whose pole strengths are to be compared. Place A on the line marked out, with its north pole pointing towards the south pole of the needle; the field of the magnet is then added to the earth's field H, and the needle oscillates quicker, say 40 per minute. We have here two fields superposed in the same direction in the region of the needle, each field being proportional to the square of the oscillations in a given time. The earth's field is then proportional to 20^2, and the two fields together proportional to 40^2; therefore the field produced by the poles of the bar magnet in the region of the needle is proportional to the difference of these squares $=40^2-20^2$. Now remove A altogether, place B in exactly the same position, and count the oscillations, say 30 per minute. In a similar manner the field in the region of the needle due to the poles of B is proportional to 30^2-20^2. Now, referring to Fig. 69A, it has been shown (para. 109) that the intensity of field due to the poles of a bar magnet at any point p on a line passing through the two poles, distant from the nearer pole say r centimetres, and from the further pole say R centimetres, is equal to

$$\frac{m}{r^2}-\frac{m}{R^2}=m\left(\frac{1}{r^2}-\frac{1}{R^2}\right) \text{ C.-G.-S. units,}$$

where m is the strength of one pole of the magnet. Now, the centre of the vibrating needle being at the point p the oscillations take place under the action of the above field. The quantity

$$\frac{1}{r^2}-\frac{1}{R^2}$$

is constant for all bar magnets of equal length placed in the same position each time oscillations are observed, and hence the magnetic fields of the two magnets A and B at the centre of the vibrating needle are proportional to their respective pole strengths, and also to the square of the number of oscillations in a given time. Hence we have

$$\frac{\text{pole strength of A}}{\text{pole strength of B}}=\frac{40^2-20^2}{30^2-20^2}=\frac{1200}{500}=\frac{2\cdot4}{1};$$

or, making the calculation by the inverse ratio of the square of the time during which a given number of oscillations are made, we should have—

20 oscillations due to earth		in	60 seconds
20 " " " "	and A	"	30 "
20 " " " "	and B	"	40 "

whence $$\frac{\text{field due to A and earth}}{\text{field due to earth}} = \frac{60^2}{30^2} = \frac{4}{1};$$

and therefore $$\frac{\text{field due to A}}{\text{field due to earth}} = 4 - 1 = 3.$$

Similarly $$\frac{\text{field due to B}}{\text{field due to earth}} = \frac{60^2}{40^2} - 1 = \frac{5}{4};$$

and, finally, the distance of the magnet poles of A and B from the needle being the same, the fields produced are proportional to their pole strengths. Hence—

$$\frac{\text{pole strength of A}}{\text{pole strength of B}} = \frac{3}{\frac{5}{4}} = \frac{2{\cdot}4}{1} \text{ as before.}$$

If the magnets are not of equal length, the distance r between the vibrating needle and the nearer pole is kept the same for each magnet, while the distance R (the magnet's length added to r) is a variable amount for each magnet. The field for each magnet is its strength of pole multiplied by the

quantity $$\frac{1}{r^2} - \frac{1}{R^2},$$

and therefore the pole strength is proportional to the number of oscillations in a given time divided by this quantity.

If the magnets whose pole strengths are to be compared are very long the term $\frac{1}{R^2}$ is very small compared to $\frac{1}{r^2}$, and may therefore be neglected, and we return to the simple proportion, as at first, that the strength of pole varies as the number of oscillations squared.

The vibrating needle must be short, to keep it in a uniform portion of the field during its movement. For near to the magnet the field is variable within very limited areas, while at greater distances it is fairly uniform throughout the area moved over by a short needle, and hence it is better to take the oscillations with the vibrating needle at some distance (say not less than 5 centimetres) from the magnet pole.

The kind of magnetic needle best suited for oscillation experiments is a short thick one—a piece of thick steel knitting needle not more than two centimetres long does very well—which can be magnetised in the ordinary way by stroking it

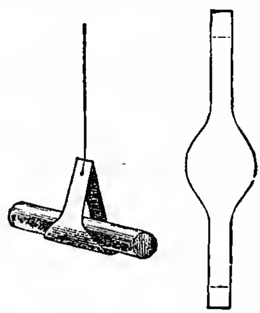


FIG. 71.

over the pole of a permanent magnet, or by wrapping a layer of insulated wire round it and passing a current through the same, tapping the needle during the operation. If it is suspended, a convenient stirrup for it may be cut from a visiting card in the shape shown in Fig. 71, bending the ends over at the dotted line, then piercing a hole through the four thicknesses of card and tying to it a fibre of unspun silk. Before the needle is put in the stirrup a somewhat heavier rod of non-magnetic metal, such as brass, should be hung therein and left for some time until all torsion is removed from the fibre. This will be the case when the rod comes to rest and remains so. The fibre should then be turned slightly at its upper point of suspension till the length of the rod lies in the mag-

netic meridian; the stirrup is then held while the brass rod is removed and the magnetic needle inserted in it with its poles the right way. The needle is then lowered till it is just above a flat horizontal surface, on which a card is placed with a black line drawn in the direction of the magnetic meridian. It is quite easy then to count oscillations every time the needle passes the black line going in a given direction. For greater delicacy a small circular mirror may be attached by beeswax to the fibre just above the stirrup, and a ray of light projected on to it from a lamp placed about two feet away, the lamp being surrounded by an opaque enclosure in which a narrow vertical slit is cut opposite the flame to emit the beam on to the mirror. The reflected ray thrown on to a white screen then shows the oscillations of the needle, and since these can be seen by this means when of very small angle, the arrangement is very sensitive, the meridian line for counting the oscillations being then marked vertically on the screen.

114. Action of a Magnetic Field on a Magnet.—It has been mentioned before that when a magnetic needle free to turn about its centre is placed in a magnetic field, it immediately sets itself in the direction of the lines of force in that field —that is, the direction of its own internal lines coincides with the direction of the lines of the field at the point where it is suspended. It is important to understand fully this action, as it immediately concerns the cause of the movement of galvanometer needles when the field by which they are surrounded is altered by starting a current in the coils.

Let us consider a magnetic needle free to move in a horizontal plane in the uniform magnetic field of the earth. It will set itself with its marked or north end towards the north pole of the earth, as in Fig. 72, where the straight lines represent the uniform lines of force of the earth's horizontal field H, and the arrow inside the magnet represents the direction of its own internal lines of magnetisation. The force acting at each end of the needle keeping it at rest in this position is equal to the product of H into the strength of the needle pole, m; that is,

the force at each end of the needle is equal to H m. This force is shown in dotted arrows at each end of the needle, and it

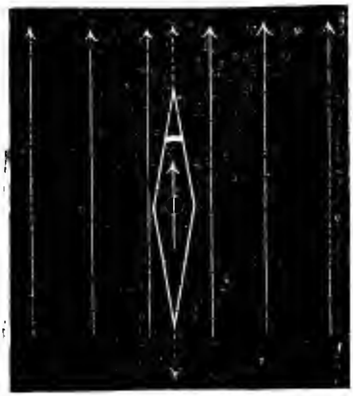


Fig. 72.

will be seen that when the needle is in this position (the meridian) there is no *turning* force exerted on it. But now,

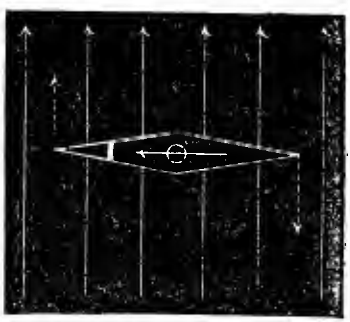


Fig. 73.

if we turn the needle through a right angle, as in Fig. 73, we shall have to exert some force to keep it there; in other

words, the force H m at each end is now tending to turn it back to the meridian.

115. "**Moment" of a Force.**—The capability of a force to produce movement, or turning about a centre, is termed the "*moment*" of that force. The moment of a force about a point is the product of that force into its perpendicular distance from the point—that is, the distance from the point to where the force is applied must be reckoned along a line perpendicular to the direction of the force. To make this clear let us take a rigid bar, A B, capable of turning about its centre, C, at one end of which a force equal to F lbs. is pulling (Fig. 74)

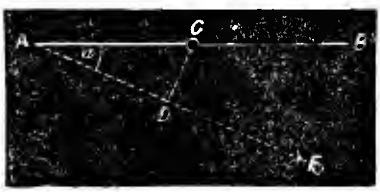


Fig. 74.

in a direction making an angle of a degrees with the bar. Let the bar be L feet long; the length A C will then be $\frac{L}{2}$ feet. Draw C D perpendicular to the direction of the force. The "moment" of the force about the point C is then—

$$F \times \text{perpendicular } C\,D.$$

Now we can find the length of C D by a simple trigonometrical ratio. (It may be mentioned here that the only trigonometrical functions of angles made use of in this treatise will be the three most frequently met with—viz., the sine, the cosine, and the tangent. The student should familiarise himself with these simple functions of angles. A table of natural sines and tangents will be required for reference. The cosine

of an angle may be found from a table of sines by subtracting the angle from 90 and taking the sine of the remaining angle.)

Now we have the ratio $\frac{CD}{AC} = \text{sine } a$; and therefore

$$CD = AC \sin a = \frac{L}{2} \sin a \text{ (feet)};$$

whence the moment of the force is equal to

$$F \times \frac{L}{2} \sin a \text{ (pound-feet)}.$$

For example, if F = 10lbs. and L = 10ft., and the angle = 30°, we have moment of $F = 10 \times \frac{10}{2} \times \frac{1}{2} = 25$ pound-feet (where $\sin 30° = \frac{1}{2}$).

It should be observed here that since $\sin 90° = 1$ and $\sin 0° = 0$ the moment of F is $\frac{FL}{2}$ when the force acts at right angles to the bar, and is *nil* when acting in the direction of the length of the bar. Hence a given force acts at the greatest advantage as regards turning a lever, or, in other words, its moment is greatest, when it acts at right angles to the direction of length of the lever. When we, for instance, exert force ourselves in order to move a heavy lever, such as the opening or closing of the gates of a river lock, we instinctively use our strength to the best advantage by pushing at the extreme end of the lever and at right angles to it. Taking the above figures with the force acting at 90deg., the moment of F becomes

$$\frac{10 \times 10}{2} = 50 \text{ pound-feet.}$$

Suppose, now, that an equal force, parallel to the first, acts at the other end, B, of the lever (Fig. 75), so as to turn it round in the *same* direction about the centre C. Drop the line C D′ perpendicular to the direction of the force,

as before. Then $\frac{\text{C D}'}{\text{CB}} = \sin a$, or $\text{C D}' = \frac{\text{L}}{2} \sin a$ (angle C B D' $= a$), and therefore the moment of the force F acting at B is

$$\frac{\text{F L}}{2} \sin a,$$

which is the same as the moment of the force acting at A. Now, as both these forces tend to turn the lever the *same* way round we must *add* their moments to obtain the moment of

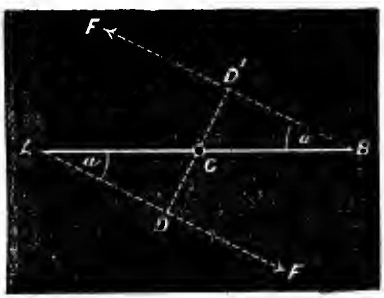


FIG. 75.

the two together, and as their moments are equal, the moment of the two will be twice that of one—that is,

$$2\left(\frac{\text{F L}}{2} \sin a\right) = \text{F L} \sin a.$$

For example, with equal forces of 10lb. each, acting at each end of the lever at an angle of 30deg., the moment of the two forces combined is

$$10 \times 10 \times \frac{1}{2} = 50 \text{ pound-feet.}$$

If the forces were acting at right angles to the lever, as in Fig. 76, the combined moment would be simply

$$\text{F L} = 10 \times 10 = 100 \text{ pound-feet.}$$

116. "Moment" of a "Couple."—When there are two equal parallel and opposite forces acting on a lever, each force tending to turn the lever in the same direction, we have what is termed in mechanics a "*couple*," and by what has been said above it will be understood that the "*moment*" *of a* "*couple*" *is equal to one of the forces multiplied by the perpendicular distance between them.*

The "moment" of the "couple" in Fig. 75 is

$$F \times \text{the perpendicular } DD' = F \times L \sin a,$$

as given above.

The "moment" of the "couple" in Fig. 76, where the equal

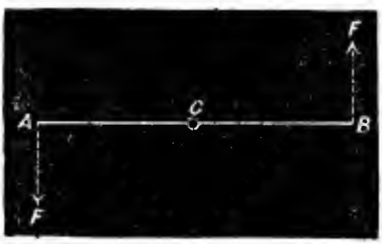


Fig. 76.

forces act at right angles to the lever, is F L. The unit "moment" in the C.-G.-S. system is the dyne-centimetre—that is, the moment of a force of one dyne acting at a perpendicular distance of one centimetre from the turning centre of a lever would be of unit value. Similarly, half a dyne acting at two centimetres distance, or ten dynes acting at one-tenth of a centimetre distance, would be of unit moment.

117. The "Couple" due to the Earth's Field.—Returning now to the magnetic needle (of strength of pole $= m$) capable of turning about its centre in the uniform magnetic field H, we know that the mechanical force acting at each end of the magnet is equal to H m dynes. These two forces then constitute a "couple," and we can now determine the moment of the couple

exerted upon the magnet in any position in which it may be placed in the field. For example, when the magnet is in the meridian (Fig. 72) there is no moment at all, although the forces are acting at each end of the magnet. There is no perpendicular distance between their directions, since they are acting in the same straight line, like the opposite sides in a "tug of war." Now, when the magnet is at right angles to the lines of force of the field (Fig. 73) the moment of the couple is a maximum, and is equal to $H m l$ dyne-centimetres, where l is the length of the magnet expressed in centimetres.

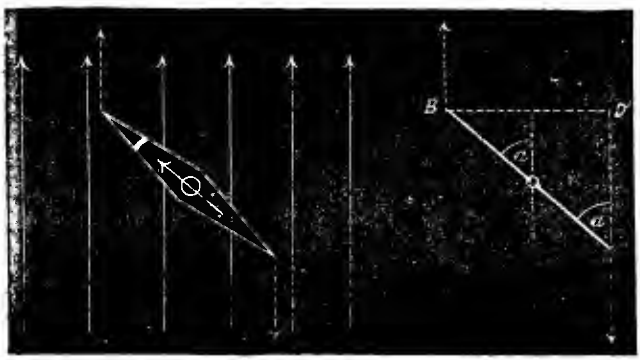


Fig. 77.

If, however, the magnet is in some intermediate position (as in Fig. 77) the moment of the couple is

$$H m \times \text{the perpendicular } B D.$$

Now, the two angles marked a are equal to each other, and indicate the angle between the needle and the meridian, and we have the relation

$$B D = l \sin a;$$

hence $$\text{moment of couple} = H m l \sin a.$$

Again, if the needle is displaced from the meridian more

than a right angle (as in Fig. 78), and we still call a the angle between the needle and meridian, we have

$$\begin{aligned}\text{moment of couple} &= \mathrm{H}\,m \times \mathrm{B\,D}\\ &= \mathrm{H}\,m\,l \sin a,\end{aligned}$$

the same as before.

Hence in whatever position the magnetic needle is placed, if we call a the angle by which it is displaced from the meridian, we have the moment of the couple tending to restore it to the meridian equal to

$$\mathrm{H}\,m\,l \sin a.$$

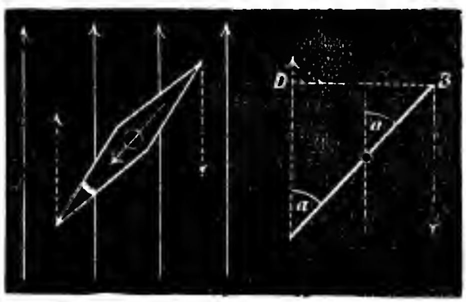


FIG. 78.

Example.—A magnetic needle whose strength of pole is 20 C.-G.-S. units and length 6 centimetres is moved 60 degrees away from the meridian; what is the moment of the couple tending to restore it to the meridian (H = ·18 C.-G.-S. units, sine of 60 deg. $= \frac{\sqrt{3}}{2}$) ?

$$\begin{aligned}\text{Moment of couple} &= \cdot 18 \times 20 \times 6 \times \frac{\sqrt{3}}{2}.\\ &= 18\cdot 66 \text{ dyne-centimetres.}\end{aligned}$$

What is the moment of the couple when the needle is 90 degrees from the meridian ?

Ans. $\cdot 18 \times 20 \times 6 = 21\cdot 6$ dyne-centimetres.

What mechanical force must be applied at right angles to this needle and *at one end* of it to keep it in equilibrium at 60 degrees?

Here the moment of the force F applied at one end is $F\frac{l}{2}$, and this must equal the moment of the couple due to the earth, which was found above to equal 18·66 dyne-centimetres.

Hence $$F \times \frac{6}{2} = 18{\cdot}66,$$

and $$F = 6{\cdot}22 \text{ dynes}.$$

If this force was applied by attaching a thread to one end of the needle, passing it over a small frictionless vertical pulley, and hanging a weight on the end of the thread, what weight would this have to be?

$$\text{Weight in dynes} = \text{mass} \times \text{acceleration of gravity}$$
$$= \text{mass in grammes} \times 981$$

and this must equal the force of 6·22 dynes.

Therefore $$\text{mass required} = \frac{6{\cdot}22}{981} = {\cdot}00634 \text{ gramme}.$$

Now the gramme mass is equal in weight to 15·43 English grains. Hence

$$\text{weight required} = {\cdot}00634 \times 15{\cdot}43 = \frac{1}{10} \text{ grain nearly}.$$

118. The "Moment" of a Magnet.—Looking at the expression, just derived, for the "moment of the couple," acting on a magnet when placed in a magnetic field, it will be noticed that the factors "m" and "l" (strength of pole and length between the poles of magnet) are properties belonging to the magnet itself, and, moreover, the moment of the couple exerted on the magnet depends directly upon the product "$m\,l$." For these reasons, the product "$m\,l$" is designated the "*moment of*

the magnet," and is usually denoted by the letter M. Putting this in the above expression, we have the moment of the couple exerted on a magnet of moment M, when placed in a uniform magnetic field H, is

$$H\ M \sin a,$$

where a is the angle between the magnet and the direction of the lines of force of the field. We are here considering the poles of the magnet as situated at its extreme ends. Although this is not strictly the case, the poles being a little nearer together than the distance between the ends, we may practically regard them as being at the ends, and not introduce any appreciable error. Whatever shape the magnet has, its moment is always the strength of one pole (m) multiplied by the distance between the poles (l), irrespective of what length of metal actually exists between the poles. For example, the moment of a semi-circular magnet is the strength of pole multiplied by the distance between the poles, viz., the diameter of the circle of which the magnet forms part. Or if we suppose a circular steel ring with a very short piece cut out, so as to form very nearly a complete circle, such as a piston ring, to be magnetised, the moment is very small, although the ring may be strongly magnetised, because the distance between its poles is very small; and if the magnetic circle be completed by inserting a piece of soft iron in the gap between the poles, there will be a continuous magnetic circuit through the mass of the metal, and the magnetic moment will be reduced to zero, because there no longer exist any free poles.

119. Two Uniform Magnetic Fields Superposed at Right Angles to Each Other. The Tangent Principle.—It is of great importance in all measurements of magnetic fields and in the measurement of electric currents by means of galvanometers to comprehend the resultant action on a magnetic needle suspended in a region where two magnetic fields, whose directions are at right angles to one another, are superposed. The following example will be easily understood. Take a bar magnet,

N S (Fig. 79), and place it horizontally at right angles to the magnetic meridian.

The magnetic field surrounding it is indicated generally by the dotted lines. It will be observed that where these lines cross the lines of force H of the earth's horizontal field we have within a limited space two magnetic fields superposed at right angles. Considering the line through the centre of the magnet, it will be noticed that near the region of the magnet the lines

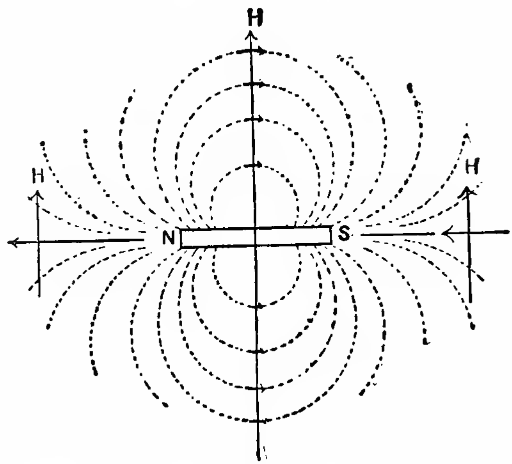


Fig. 79.

of force form small circles, and therefore that portion of a circle which crosses the earth's field H can only be regarded as straight or uniform for a very short length. As we take points further removed from the magnet, however, the circles of force are larger, and that portion of them which cuts H may be taken as straight or uniform for a greater length. At greater distances away from the magnet, however, the intensity of the field becomes lessened.

Now at any given point in this magnetic field there is a *resultant direction* of the lines of force and a *resultant intensity*,

which is, in fact, the resultant of the two forces due to the earth and the magnet acting on unit pole at that point. We shall, however, for the present only consider the resultant intensity and direction of the field at points where the two component fields may be considered at right angles to each other. Let us consider a unit pole placed at the point o (Fig. 80) under the influence of two magnetic fields of H and f units of intensity (para. 109 and 111). Now, these two fields will act on the unit pole with forces equal to H and f dynes

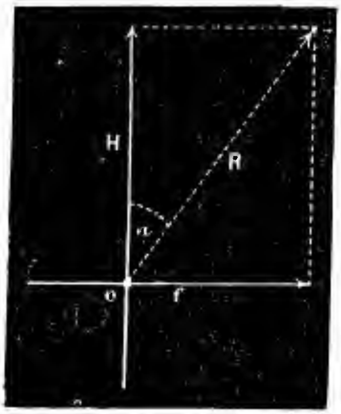


FIG. 80.

(para. 114) in their respective directions, and we may find their resultant in direction and magnitude by the parallelogram of forces. Let the length of the lines H and f be proportional to the respective strengths of the two forces. On completing the parallelogram we get the resultant force R by drawing the diagonal, and if we can determine the angle a, we shall ascertain the *direction* of this resultant. From the parallelogram it is seen that

$$\tan a = \frac{f}{\mathrm{H}},$$

and, therefore, the strength of the two fields, or the relation

between them, being known, we can determine the direction of the resultant field. This is, in fact, the direction of the lines of force due to the two fields H and f, and a freely suspended magnetic needle will always set itself in the direction of the lines of force of a field when the latter is uniform—that is, when the lines of force are parallel in the region of the needle, which is what we are now considering. If the lines of force are in curves, the needle will set itself as a tangent to the curve. Experimentally, therefore, we may observe this resultant direction by setting up a horizontally-suspended magnetic needle, and first allowing it to come to rest under the influence of one of the fields, say H, the earth's horizontal field, and then bringing into action the superposed field f by placing the magnet N S (Fig. 79) at right angles to the meridian line on which the magnet is suspended. The tangent of the angle moved through by the needle is then equal to the ratio of the two fields, *i.e.*, f : H, and if we know the strength of one of those fields, say that of H, we can find that of the other (f) by

$$f = \mathrm{H} \tan a.$$

This is the tangent principle which comes so frequently in use in the measurements of magnetic fields; not only those fields due to permanent magnets, but also electro-magnets, solenoids, and galvanometers. We shall, therefore, refer again to this principle, as proved above, when we have these other magnetic fields under consideration.

Example.—A very short magnetic needle is suspended by a long torsionless silk fibre, so that it is free to move in a horizontal plane. A long light non-magnetic pointer is attached to it, the end of which moves over a scale graduated in degrees, by means of which the angle moved through by the needle may be observed. As soon as the needle has come to rest in the earth's magnetic meridian, a bar magnet is placed at a given distance from the centre of the needle, its position being such that its length is bisected by and is at right angles to the meridian line through the needle. The latter is then observed by the pointer to move through 60 degrees. What

is the intensity of the field (f) at the centre of the needle due to the bar magnet?

Answer.
$$f = \text{H} \tan 60$$
$$= \cdot 18 \times 1\cdot 7321 = \cdot 31177 \text{ C.-G.-S. units.}$$

Similarly, the intensity of the resultant field may be determined from the same parallelogram (Fig. 80), from which it is evident that

$$\text{resultant field (R)} = \frac{\text{H}}{\cos a} = \frac{f}{\sin a},$$

the intensity of which in C.-G.-S. units may, therefore, be found when the strength of one of the fields and the direction of the resultant field, or the angle a of deflection, are known.

Example.—In the last example what is the intensity of the resultant field—

(1) Having given that H = ·18 C.-G.-S. units?

Answer. $\text{Intensity} = \dfrac{\text{H}}{\cos 60} = \dfrac{\cdot 18}{\cdot 5} = \cdot 36$ C.-G.-S. units.

(2) Having given that $f = \cdot 31177$ units?

Answer. $\text{Intensity} = \dfrac{f}{\sin 60} = \dfrac{\cdot 31177}{\cdot 866} = \cdot 36$ C.-G.-S units.

It follows, therefore, that the lines of force surrounding a magnet cannot be symmetrical throughout, since they are really the resultant lines of two fields, that of the magnet and that of the earth. If we only consider the horizontal field of the earth and lay the magnet down horizontally, say at right angles to the meridian, as in Fig. 79, we should not expect to find its magnetic field perfectly symmetrical about its own figure, as drawn in that diagram. When, however, the field is examined by the iron filing method it does appear quite symmetrical, simply because the want of symmetry is only apparent *outside* the area of field made visible by the filings. What the filings reveal to us is really the resultant field, but the earth's field is so weak in comparison to the field in

close proximity to the magnet, that the resultant direction is practically that of the magnet's field. It is interesting, however, to detect by a somewhat more delicate means the state of the field from points near the magnet up to distances so far away that the field is no longer apparent. This may be done very well by observing the angles of deflection of a small compass needle placed in succession at various parts of the field. In Fig. 81 is shown a field so analysed along one direction, viz., the earth's meridian. The small arrows represent the approximate direction of the compass needle when placed at those points, the arrow heads being the needle's north pole, and therefore representing by the direction of the arrows the direction of the lines of force. At a distance from the magnet

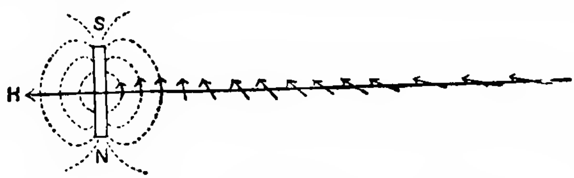


FIG 81.

equal to about six times its length, the needle took up, as nearly as could be observed, the direction of the meridian, showing that the field of the magnet was at this point practically *nil*. The dotted curves close to the magnet N S cover about the area of the field usually indicated by iron filings, the field appearing symmetrical within these limits. A line was drawn on a flat surface corresponding with the magnetic meridian, and divided out into inches. The magnet was then laid down at right angles to the line, being bisected by it. The small compass needle was then placed at each inch mark along the line, and the directions taken up by the needle noted down as nearly as possible as shown in the figure. At the point where the compass needle stood at 45 degrees the two superposed fields were equal, because tan $45° = 1$, and therefore $\frac{f}{H} = 1$.

120. The Magnetic Field of a Magnet in Terms of its Moment.—Let us suppose a unit north pole at the point n (Fig. 82) on a line bisecting a bar magnet, N S, whose length is l centimetres and strength of each pole m C.-G.-S. units. The unit pole will be acted upon by two forces, one of attraction and one of repulsion, in the direction of a line drawn from S to n and N to n respectively. The magnitude of each of these forces will be $\frac{m}{r^2}$ (para. 108), r being the distance between the

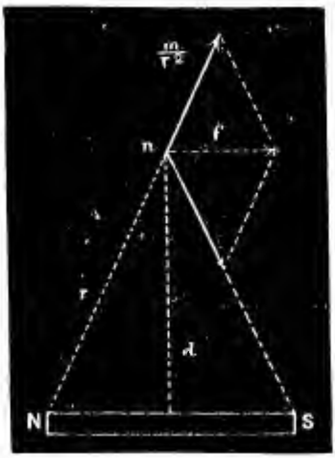


FIG. 82.

unit pole and either pole of the magnet. Now, the resultant of these two forces will be the actual force on the pole, and will be numerically equal to the intensity of field (f) at the point n. On completing the parallelogram, and drawing the resultant f we find by the simple proposition of similar triangles that

$$f : \frac{m}{r^2} :: l : r;$$

whence $$f = \frac{m\,l}{r^3};$$

and since $m\,l$ is the moment (M) of the magnet (para. 118),

$$f = \frac{M}{r^3} \text{ in C.-G.-S. units.}$$

Example.—(1) What is the intensity of the magnetic field of a bar-magnet at a point 20 centimetres distant from either pole, the moment of the magnet being 800 dyne-centimetres?

Answer.—Intensity $= \frac{800}{20^3} = \frac{1}{10}$ C.-G.-S. unit.

(2) What is the intensity of the magnetic field due to a bar magnet at a point 30 centimetres away from the centre of the magnet in a direction perpendicular to its length, the strength of each pole of the magnet being 900 C.-G.-S. units and its length 20 centimetres?

Here the direct distance (r) from either pole to the point in the field is not given, and must be calculated from the data given. It will be seen (Fig. 82) that the distance r forms the hypothenuse of a right-angled triangle, of which the other two sides d and $\frac{1}{2}\,l$ are known, viz., 30 and 10. The length r is therefore the square root of the sum of the squares of the other sides (Euclid I., 47)—that is,

$$r = \sqrt{30^2 + 10^2} = \sqrt{1000} = 31{\cdot}62 \text{ centimetres,}$$

and the moment of the magnet $= m\,l = 900 \times 20 = 18{,}000$ dyne-centimetres. Therefore the intensity of the field at the required point is

$$\frac{M}{r^3} = \frac{18000}{(31{\cdot}62)^3} = \frac{18000}{31620} = {\cdot}569 \text{ C.-G.-S. units.}$$

If the point at which the field is calculated is, at a perpendicular distance from the magnet, equal to many times its length, we may take the hypothenuse (r) or pole distance from the point as being equal to the actual perpendicular distance, and obtain a very near approximation. This could not be done in the

last example without introducing an error of 17 per cent., the ratio of the perpendicular distance to the length of magnet being only as 3 : 2.

Let us now measure the field at a point situated on the continuation of the axis of the magnet, d centimetres from its centre, r centimetres from the nearer pole, and R centimetres from the further pole, supposing that a unit north pole is placed at n at the point required (Fig. 83). It is evident that the two poles of the magnet act oppositely on the unit pole, the nearer pole (N) having the greater effect, and exerting a force of repulsion equal to $\frac{m}{r^2}$ dynes on the unit pole.

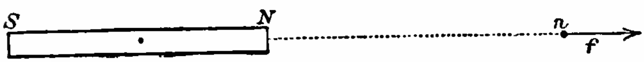


FIG. 83.

The other pole (S) acts with a force of attraction equal to $\frac{m}{R^2}$ dynes, and the net or resultant force acting at n will be the difference of these two, viz. :—

$$\frac{m}{r^2} - \frac{m}{R^2},$$

this being also equal numerically to the intensity (f) of field at the point. The *direction* of the resultant is *from* the magnet in a line with its axis in the diagram. If the magnet were reversed the *direction* of the resultant would be reversed, its intensity remaining the same.

Now, $$r = d - \frac{l}{2},$$

and $$R = d + \frac{l}{2}.$$

Putting these values in the numerator of the above, we obtain, after cancelling,

$$f = \frac{2\,m\,l\,d}{r^2\,R^2},$$

where $m\,l$ is the moment (M) of the magnet. Therefore,

$$f = 2\,\mathrm{M}\,\frac{d}{r^2\,\mathrm{R}^2}.$$

Example.—What is the strength of field at a point on the continuation of the axis of a magnet 30 centimetres (d) from its centre, length of magnet 20 centimetres, and pole strength 900 C.-G.-S. units?

Moment of magnet	$= 900 \times 20 =$	18,000 units.
Distance from nearer pole (r) $=$	$30 - 10 =$	20 cms.
„ „ further „ (R) $=$	$30 + 10 =$	40 „

Hence,

$$\text{Intensity of field} = 2 \times 18{,}000 \times \frac{30}{20^2 \times 40^2}$$

$$= \frac{36 \times 3}{64} = 1{\cdot}68 \text{ C.-G.-S. units.}$$

Similarly, we may arrive at an approximate value of the field by a little simpler process (when the point is distant from the magnet many times its length) by reducing the formula to the following form (substituting the values of r and R and reducing by simple algebra),

$$f = \frac{2\,\mathrm{M}}{d^3\left(1 - \dfrac{l^2}{4\,d^2}\right)^2}$$

from which it is evident that when d is great compared to l, the quantity within the bracket approaches unity, and we have

$$f = \frac{2\,\mathrm{M}}{d^3}.$$

Example.—What is the strength of field at a point situated 2 metres away from the centre of the magnet in the last example? Here we may adopt the approximation, as the quantity in the bracket is close upon unity, and we have

$$\text{Intensity} = \frac{2 \times 18{,}000}{(200)^3} = {\cdot}0045 \text{ C.-G.-S. units.}$$

121. Polar and Normal Lines of a Magnet.—In a similar manner the magnetic intensity at points in the field lying in other directions may be found by the application of the parallelogram of forces and geometrical principles; but the two directions in which we have investigated the strength of field are those in which measurements of the magnet and its field are taken, and, therefore, are enough for our purpose.

For conciseness we shall hereafter designate these two directions the *polar line* and the *normal line* of a magnet. The

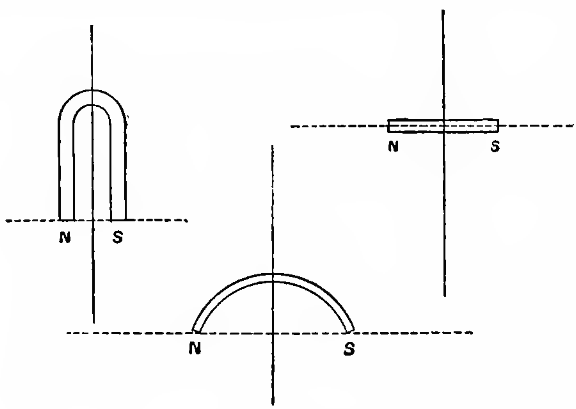


FIG. 84.

polar line will be understood to mean a straight line of indefinite length passing through the two poles of a given magnet, while the *normal line* will mean a straight line of indefinite length at right angles to the polar line, and which also bisects the distance between the two poles. In Fig. 84 three common forms of magnets are shown, their polar lines being dotted, and normal lines shown plain.

122. The Magnetometer.—We have already seen (para. 119) that when two magnetic fields of intensities f and H are super-

posed at right angles to one another at a given point, the ratio $\frac{f}{H}$ is equal to the tangent of the angle between the direction of the field H and the resultant field—that is, the tangent of the angle moved through by a short magnetic needle from its position of rest in the field H. To measure this ratio, then, we require a suspended magnetic needle moving over a scale of degrees by which its movement may be observed. The conditions which are required in such an instrument are

1. Short, well-magnetised steel needle.
2. Long light pointer or mirror attached to same.
3. Long unspun silk fibre to suspend needle and pointer (the length being required to eliminate error due to stiffness or rigidity of fibre).
4. Glass covering over needle and suspension to avoid disturbance by currents of air.
5. Scale divided into natural tangent divisions.
6. Means of accurately levelling.

An instrument for the above purpose is known as a "magnetometer," and the student will find it an excellent introduction to the theory and use of galvanometers to construct a magnetometer of the simple design here described, and carry out some measurements of magnetic fields with its aid. The following will be found quite simple to make:—A glass funnel, F, of thin glass (Fig. 85) stands inverted on a flat wooden baseboard, B, which should be provided with levelling screws; or a glass flask, with its lower half neatly cut off, answers the same purpose. Through the cork in the neck at the top is pierced a piece of No. 12 copper wire, bent into a ring at its upper end, so that it may be turned round with the finger and thumb, the lower end being filed flat, and a small hole drilled, to which the silk suspension may be tied (Fig. 86). This piece of wire should fit tight in the cork, so that when it is once adjusted it will not slip. The lower end of the silk is then tied to the stirrup. To make the latter, stiff note paper does very well, cut to the shape in Fig. 87, the ends being

folded over as shown to make it stronger, where holes are pierced to which to tie the silk. The three lines across the centre indicate where it should be folded to form a narrow groove at the lower part of the stirrup in which to place the long pointer. This ensures the pointer being in the same line as the needle. The diagram also shows the needle and pointer as mounted in the stirrup. A more durable stirrup may be made in the same way with copper or lead foil. Before mounting the needle and pointer

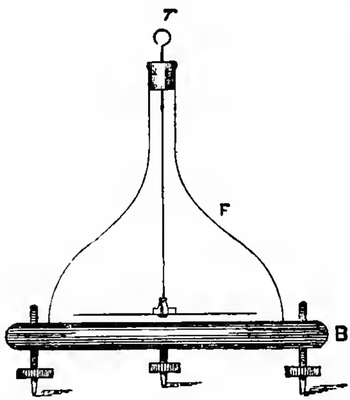


FIG 85.

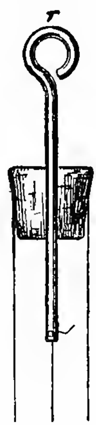


FIG. 86.

a brass or copper rod about 6in. long should be inserted in the stirrup, balanced and left for some time till it comes to rest. During this operation the inverted funnel may be lifted off its stand and placed with its edges resting on two supports, so that the hands may conveniently reach underneath to the lower end of the fibre. This process will remove all twist from the fibre. Now the top of the fibre should be turned by turning the ring *r* until the length of the brass rod lies in the magnetic meridian, ascertained by its direction coinciding as nearly as possible with that of a compass needle

placed close to the instrument. The brass rod may now be removed by the right hand, while the stirrup is held in the left to prevent it turning, and then the long pointer placed in the lower groove and balanced, and the short magnetic needle placed above it.

The magnetic needle should be about a centimetre long, but not more than $1\frac{1}{2}$ centimetre, and of knitting-needle steel. It requires a little practice to draw out a good, straight, light pointer.

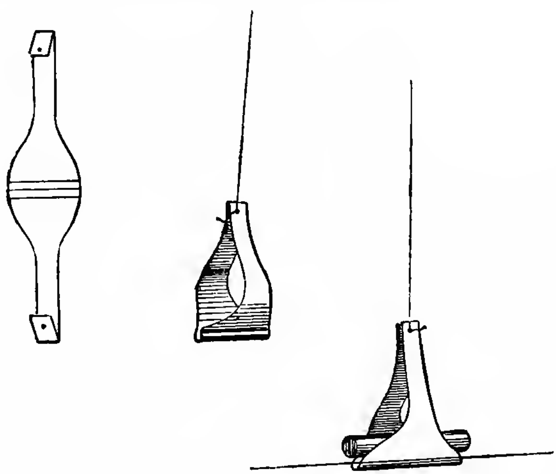


FIG. 87.

Some *thin* glass tubing, about $\frac{3}{16}$-in. bore, should be procured, and a piece held by its ends over a Bunsen burner or spirit lamp flame with the two hands, turning it round to heat it evenly in one place till about three-quarters of an inch of it is perfectly red hot, and evenly hot all round. Then removing it from the flame the two ends are pulled, at first quickly, and then slowly, which draws out a fine capillary tube two or three feet long. After pulling out, the fine tube must be kept stretched tight till cool, for if the pull at the ends is relaxed immediately after

pulling out, the tube will not cool perfectly straight. The straightest and lightest portion of the drawn tube must now be snipped off to the required length—say, five or six inches—and is then ready for use.

A more elaborate instrument fitted as a magnetometer is shown in Fig. 88, in which the suspension can be controlled

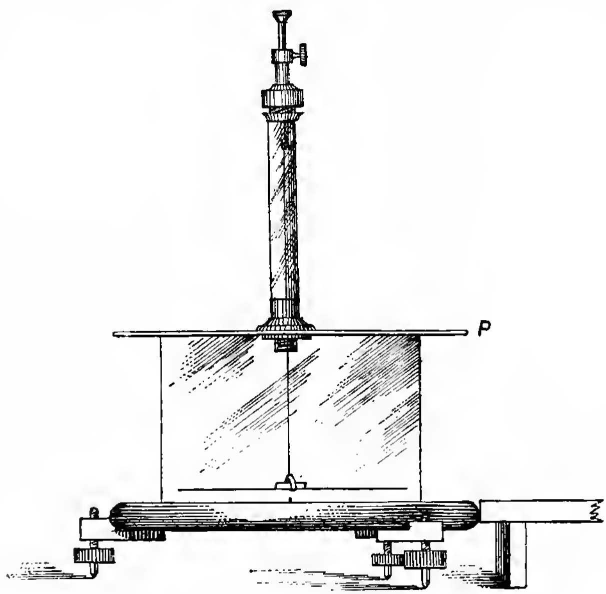


FIG. 88.

with more certainty. The details of the suspension head are represented in Fig. 89, where a tubular brass cap, C, screw-threaded, is cemented on to the upper end of the glass tube T. Above this is a brass tube screwing on at its lower end to the cap C, and carrying at its upper end a split ring, R, through which passes the brass rod B supporting the suspension. The latter may therefore be turned round, raised or lowered

as required, and then clamped fast by the screw on the split ring R. The glass tube at its lower end has also a tubular brass cap cemented to it and screw-threaded, a nut on the underside of the circular glass plate P clamping it to the same. A table about 2ft. long and the same height as the baseboard of the instrument will be required for placing magnets on whose fields are to be measured.

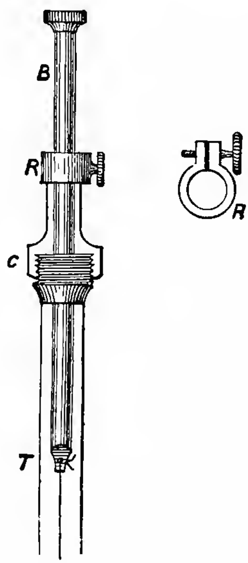


FIG. 89.

A graduated scale must now be made, by which the movement of the pointer, when the needle is deflected, may be observed. If this is marked out in degrees, the *angular* deflection of the needle will be observed, and the corresponding tangent must be ascertained from a table of tangents. It is better, however, to graduate the scale in divisions proportional to natural tangents, the method of doing which will now be given

123. Graduation of Tangent Scale.—It has already been shown (para. 98) that in a figure such as Fig. 90 distances measured from A along the line A B are proportional to

FIG. 90.

the tangents of the angles which those distances subtend. For example, two equal angles are drawn at C (Fig. 90); the tangent of the first angle is proportional to A D and that

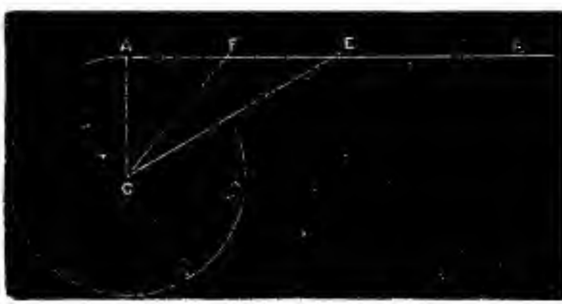


FIG. 91.

of the two together to A E. It is evident, that when the force by which a magnetic needle is deflected (from a zero position at A C) is proportional to the tangent of the angle of deflection, it requires *more than* twice the force to move the

needle through twice the angle, for A E is more than double A D. The object, then, of dividing out a scale in tangent divisions is to make the divisions proportional to the force moving the needle. This principle is shown in Fig. 91, where the distance A E is divided into two equal parts at the point F. Now, on drawing F C the scale is divided into divisions proportional to the force exerted on the needle; that is, the force which would move the needle from the direction A C to that

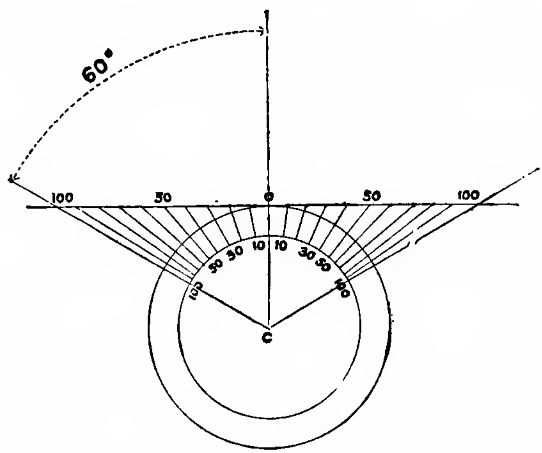


Fig. 92.

of C E would be just double that required to move it from A C to C F, the needle turning, of course, about the centre C. The method is therefore to mark out a number of equal divisions on the line A B, starting say from A, and then to draw lines from these divisions to the centre C, dividing the circular scale into divisions proportional to the tangents of the respective angles, and therefore proportional to the forces exerted on the magnetic needle. It only remains to decide as to the size of each division on A B. In the case of tangent galvanometers,

the tangent scales of which are generally divided out up to 60°, the line C E is drawn, making an angle of 60° with the vertical A C. The horizontal distance A E is then marked out into 100 equal divisions, and the circular scale is graduated by drawing lines from these equal divisions towards C. This process is shown in Fig. 92, where it will be seen that the divisions become very crowded near 100, and it is useless to carry them beyond 60°; therefore it is usual to divide the scale on the opposite side of the circle in degrees of arc up to 90° on each side, by which means angles of

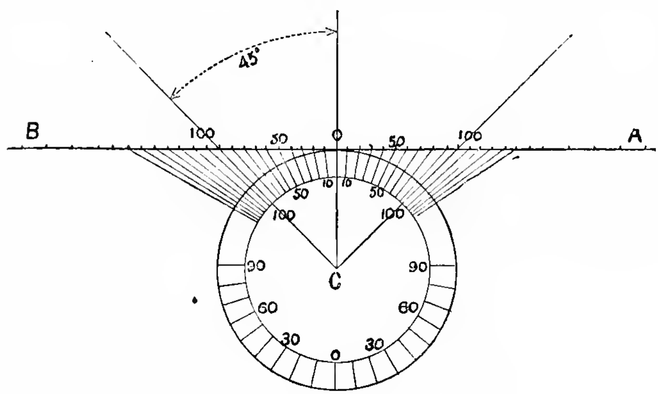


Fig. 93.

deflection higher than 60° may be read off on the degree scale, and the tangents found by reference to tables. It is also customary for the simple comparison of currents, where the natural tangents are not required, to read from the tangent scale; but for the *measurement* of a current, to read from the degree scale, and find the natural tangent of the angle by the tables (see example in para. 100). It will be found, however, very convenient, when the scale of the magnetometer is fairly large, say not under six inches diameter, to select 45° as the starting point (Fig. 93), and to divide into 100 equal parts the distance which this angle cuts out on A B. We have

then the advantage that any scale reading divided by 100—that is, moving the decimal point two places to the left—gives at once the natural tangent of the angle, for the reason that the tangent of 45° is unity. For instance, if we had a deflection of 85 divisions, the natural tangent of the angle corresponding to the deflection is ·85 and the angle itself need not be known The line A B is marked out on both sides up to about 60° or 70°, the length of each division being the same all along. A straight rule is then placed in line between each division in succession and the centre, and the corresponding scale lines drawn at the edge of the scale circle for each position. The

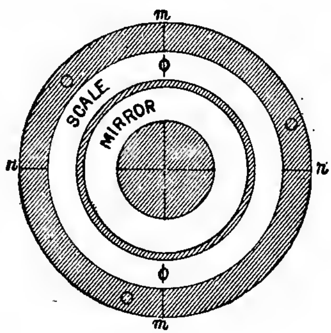


FIG. 94.

lines may be drawn from each single division near zero, but as the divisions become closer together they may be drawn from every second or fifth division, the intermediate divisions being judged by the observer when taking readings. The other side of the circle should then be marked out in degrees, up to 90° on each side.

124. Adjustment of Magnetometer.—To complete the magnetometer and set it up for use the wooden baseboard should have two fine lines *m m* and *n n* (Fig. 94) cut or marked on it accurately at right angles to each other and passing through the centre ; a circle should also be described

on the board a trifle larger in diameter than the outside diameter of the scale, to act as a guide in placing the scale symmetrically with the centre. The scale is cut out ring shape and placed in position with its zero marks in line with the line *m m* on the baseboard. It is better to let the scale lie loose on the board, as once placed there is nothing to move it, and sticking it down causes bulging. Inside the scale there should be a ring of mirror glass as shown in the figure; this is for the purpose of reading accurately the position of the pointer, the reflection of which is seen in the mirror. At the centre of the baseboard there should be fixed a little brass point standing vertically, only to the height of about $\frac{1}{8}$ inch above the level of the board; this will be found very convenient for levelling the instrument, during which operation the eye may be kept on a level with the surface of the board, and the levelling screws turned till the centre of the suspended needle is seen to hang immediately above the centre brass point, viewed from all sides. The previous directions having been carried out for the removing of torsion from the fibre, the baseboard is placed with the line *m m* as nearly as possible in the magnetic meridian, and the inverted funnel carrying the suspended needle and pointer is then placed on the board, the rim of the same being placed symmetrically with the scale. The height of the needle can now be adjusted by raising or lowering, not turning, the wire suspension head; the stirrup should then just clear the brass centre point. It will most likely be found that the pointer is very near zero, but not quite. To bring it to zero, the baseboard of the instrument must be turned round, little by little, till the pointer is at zero. This adjustment is somewhat tedious, because the needle must be allowed to come to rest between each adjustment, and the slightest movement of the board often sets the needle swinging considerably, and alters the levelling.

This inconvenience may be dispensed with in the following way: Mount the ordinary circular baseboard B (Fig. 95) on a second and larger board, B^1, countersunk to receive it. Attach the levelling screws to the lower board, and let the glass shade carrying the suspended needle rest on this.

Through the centre of the upper baseboard fix a brass spindle, so as to enable the board to be turned by means of the lever handle H. This lever should not be rigidly fixed to the spindle unless made short enough to work inside the levelling screws. It may be made like a spanner and fit on to a nut head at the lower end of the spindle, and can then be removed when once the instrument is adjusted. The scale and mirror are on the upper board, so that these may be turned till the zero point is exactly under the pointer, while the needle itself and the levelling of the instrument are not disturbed. The brass spindle might also carry the centre point, as shown. Without this elaboration, however, once the adjustment is made it will never require repeat-

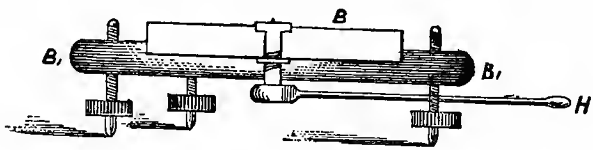


FIG. 95.

ing, if the instrument is not moved. No magnets or pieces of iron must be anywhere near the instrument during this adjustment.

125. **The Moment of a Magnet Measured by its Field.**—The two methods, due to Gauss, by which the moment of a magnet may be determined can now be easily understood. By the first method the magnet is placed so that its normal line (para. 121) is in the same magnetic meridian as the magnetometer needle, the distance between the centres of the needle and magnet being known. In Fig 96 let the circle represent the magnetometer in plan, properly mounted and adjusted. A wooden table, A, is placed close to the magnetometer board, its surface being marked out in cross lines indicating distances in centimetres from the centre of the suspended needle, and its height such that magnets placed on it are in the same horizontal plane

as the needle. The line drawn longitudinally along the centre of the table must lie in the magnetic meridian, and be the continuation of the line joining the zero points on the scale (the line *m m*, Fig. 94); the suspended needle will then lie in the same line before any magnet is placed on the table.

Now let a magnet, N S (Fig. 96), whose moment is to be determined, be placed on the table so that its polar line agrees with one of the cross lines at a certain distance from the needle, and its normal line coincides with the central line along the table. The magnetometer needle is then deflected, and the natural tangent of the angle of deflection is the scale reading divided by 100 (para. 123). Now, in the region of the needle we have two fields (H of the earth and f of the

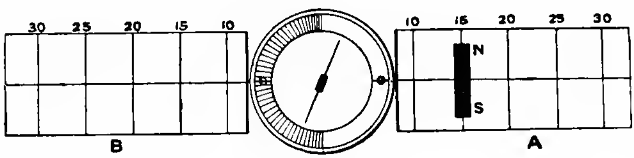


FIG. 96.

magnet) superposed at right angles to each other, and by para. 119 the ratio of f to H is the tangent of the angle a between H and the resultant field. That is,

$$f = \mathrm{H} \tan a \text{ (in C.-G.-S. units),}$$

the angle a being that moved through by the magnetometer needle from its position of rest in the field H. Also the intensity of field on the normal line of a magnet at a point r centimetres from either pole has been shown to be the moment of the magnet divided by r^3 (para. 120); that is—

$$f = \frac{\mathrm{M}}{r^3} \text{ (in C.-G.-S units).}$$

From these two values for f we obtain the relation

$$\frac{\mathrm{M}}{r^3} = \mathrm{H} \tan a;$$

whence $$\mathrm{M} = r^3 \mathrm{H} \tan a.$$

The only observations to be made, then, are the tangent of the angle of deflection and the distance r. Knowing the distance d of the magnet from the needle, we have (para. 120)

$$r^2 = d^2 + \left(\frac{l}{2}\right)^2;$$

whence

$$r^3 = \left[d^2 + \left(\frac{l}{2}\right)^2\right]^{\frac{3}{2}},$$

where l is the length of the magnet.

In order to eliminate error in placing the magnet or in observing the deflection, another observation is taken with the magnet reversed on table A giving the deflection on the opposite side of zero. Again the table is moved to the other side B, where two similar readings are taken. It is better to have two tables, carefully adjusted and fastened down in position so that a number of observations may be taken of different magnets without altering that adjustment. All screws or clamps used in the making of the tables or securing them in position must be of brass, not of iron.

The sum of the four readings so taken divided by four gives the mean deflection, which is then used in the calculation.

Example.—(1.) Determination of the moment of a galvanometer controlling-magnet (Fig. 65). Mean of four observations, 60 divisions on the tangent scale, distance between centres 30 centimetres. Whence tan $a = \cdot 6$,

$$r^3 = (30^2 + 7 \cdot 5^2)^{\frac{3}{2}} = (956 \cdot 25)^{\frac{3}{2}},$$

and H being $\cdot 18$ C.-G.-S. units, we have

$$\text{Moment} = (956 \cdot 25)^{\frac{3}{2}} \times \cdot 18 \times \cdot 6.$$

This is most readily worked by logarithms, thus:—

$$\log\,(956 \cdot 25)^{\frac{3}{2}} = \frac{2 \cdot 9806 \times 3}{2} = 4 \cdot 4709$$

$$\log\,\cdot 18 \qquad = \bar{1} \cdot 2553$$

$$\log\,\cdot 6 \qquad = \bar{1} \cdot 7782$$

$$3 \cdot 5044 = \log 3194.$$

The required moment is therefore 3,194 dyne-centimetres, and the distance between the poles of this magnet being 15 centimetres, the strength of each pole is

$$\frac{M}{l}=\frac{3194}{15}=213 \text{ C.-G.-S. units.}$$

(2.) A compound horseshoe magnet (Fig. 62), consisting of 8 separate magnets and measuring 8·8 centimetres between the centres of its poles, gave a mean of 131 divisions. Distance between centres 35 centimetres. Its moment was therefore—

$$(35^2 + 4{\cdot}4^2)^{\frac{3}{2}} \times {\cdot}18 \times 1{\cdot}31 = 10{,}351 \text{ dyne-centimetres.}$$

$$\text{and strength of pole} = \frac{10351}{8{\cdot}8} = 1176 \text{ C.-G.-S. units.}$$

(3.) A powerful steel bar magnet 1 foot in length (30·48 centimetres), gave a mean of 87·5 divisions. Distance between centres 45 centimetres. Its moment was therefore—

$$(45^2 + 15{\cdot}24^2)^{\frac{3}{2}} \times {\cdot}18 \times {\cdot}875 = 16{,}890 \text{ dyne-centimetres,}$$

and strength of pole

$$=\frac{16890}{30{\cdot}48}=554 \text{ C.-G.-S. units.}$$

(4.) A similar sized magnet under the same conditions gave a mean of 83·25 divisions. Hence its moment (by comparison with the preceding) was—

$$16890 \times \frac{{\cdot}8325}{{\cdot}875} = 16{,}070 \text{ dyne-centimetres,}$$

and strength of pole

$$=\frac{16070}{30{\cdot}48}=527 \text{ C.-G.-S. units.}$$

By the second method of Gauss the magnet is placed so that its polar line passes through the centre of the magneto-

meter needle at right angles to the meridian, the distance between the centres of the needle and magnet being known.

The table must therefore be placed at right angles to its former position, as in Fig. 97, in which the magnetometer needle is shown deflected by a magnet. One reading is taken with the magnet placed as shown, a second reading with the magnet reversed, and two similar readings with the table placed on the other side of the magnetometer, the mean of the four being finally taken.

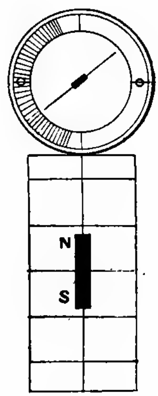


FIG. 97.

Now, in para. 120 it was shown that the intensity of field f at a point on the polar line of a magnet situated d centimetres from its centre was expressed by

$$f = \frac{2\,M}{d^3\left(1 - \frac{l^2}{4\,d^2}\right)^2},$$

and in para. 119 the ratio of two magnetic fields of intensity f and H superposed at right angles to each other was shown to be equal to the tangent of the angle a between H and the resultant; that is—

$$f = H \tan a\,;$$

whence, equating these two values of f,

$$\text{magnetic moment (M)} = \mathrm{H}\frac{\tan a}{2} d^3\left(1 - \frac{l^2}{4\,d^2}\right)^2.$$

Example.—(1.) A small bar magnet, 10·3 centimetres long, gave a mean deflection of 105·5 divisions, the distance between centres being 20 centimetres. Its moment was therefore

$$\cdot 18 \times \frac{1\cdot 055}{2} \times 8000\left(1 - \frac{106\cdot 09}{1600}\right)^2$$

$$= \cdot 18 \times \cdot 5275 \times 6978\cdot 8$$

$$= 662\cdot 6 \text{ dyne-centimetres.}$$

(2.) The same magnet was placed with its centre at 30 centimetres distance from the centre of needle. The mean deflection was then 29 divisions; the moment was therefore

$$\cdot 18 \times \frac{\cdot 29}{2} \times 27000\left(1 - \frac{106\cdot 09}{3600}\right)^2$$

$$= \cdot 18 \times \cdot 145 \times 27000 \times \cdot 9419$$

$$= 663\cdot 7 \text{ dyne-centimetres.}$$

This last was taken to check the former reading, the mean of the two results being 663 dyne-centimetres, and

$$\text{strength of pole } \frac{663}{10\cdot 3} = 64\cdot 3 \text{ C.-G.-S. units.}$$

126. **The Nature of Magnetism.**—At the point now arrived at, it may be of interest to discuss briefly what is the difference in the internal condition of a piece of iron or steel before and after magnetisation. In the first place, it has long been admitted that magnetism is *molecular* in its nature, which is evident from the fact that, however we subdivide a magnetised steel rod, we have in every portion so subdivided a perfect magnet, with two poles of opposite name; and this is true, however far we carry the subdivision. So that if we conceive the subdivision carried out until the molecule itself is reached, we should expect to find it a perfect magnet.

It is only by the generalisation of experimental researches, and in the endeavour to assign reasons for, and explanations of, the effects and phenomena we observe in Nature, that her laws are discovered and real progress made in any branch of science. A consistent and logical explanation of a set of observed effects is known as a "theory" of those effects. For a theory to be a true one it must explain *all* the observed effects and phenomena, and hence it sometimes happens that a new experiment or fact observed modifies or even upsets a previously accepted theory. But a theory, though imperfect, answering all the phenomena then known, is decidedly better than none at all, for some of our most reliable and adaptable theories at the present time have been the outcome of a long process of evolution from previously held theories.

In magnetism the observed effects to be explained might be summed up in the following questions:—

1. What is the condition of the molecules in a mass of iron or steel before the mass is magnetised?

2. What is their condition after the mass is magnetised?

3. On ceasing the process of magnetisation of the mass why is the polarity retained when the mass is of steel, and almost completely lost when it is of soft annealed iron?

4. Why do blows from a hammer and mechanical strain or stress assist the process of magnetisation when the mass is in the presence of an inducing cause, and facilitate the disappearance of magnetisation when the inducing cause is removed?

5. Why does heat produce the same effect?

6. What is the cause of the elongation of iron when magnetised?

7. What is the reason for the audible sounds emitted by iron during its magnetisation?

8. If we take a magnet and de-magnetise it until there is no evident magnetism left, is the condition of the molecules the same as that previous to magnetisation?

Those who were present at the Society of Telegraph-Engineers and Electricians on the 24th May, 1883, when Prof. D. E. Hughes, F.R.S., read a Paper on "The Cause of Evident

Magnetism in Iron, Steel, and other Magnetic Metals," will not easily forget the enthusiasm with which the author's new theory of magnetism therein contained was received. Perhaps the greatest claim to general acceptance, and certainly the charm most attractive to his hearers, lay in the fact that Prof. Hughes completely demonstrated his theory experimentally. This, in itself, was a refreshing contrast to the necessity for imagination required in grasping the previous theories of Coulomb, Poisson, and Ampère; the two former of whom supposed the molecule to be spherical, containing two opposite magnetic fluids, which were separated when magnetised, and mixed when demagnetised or neutral, and that a peculiar force, called coercive force, present in molecules of steel, retained the evident magnetism by preventing the involuntary mixing of the two fluids. Of this there was no experimental demonstration. Ampère supposed each molecule to be an electro-magnet; that is, an electric current was supposed to be constantly flowing round it, and that under magnetisation the molecules were symmetrically arranged, while in a neutral state they were arranged in haphazard directions. Besides other objections to this theory we have the anomaly of assuming a coercive force in soft iron to cause the rotation of at least half the molecules in order to bring about neutrality. De la Rive, Weber, and Maxwell made a step in advance by propounding the *inherent polarity or magnetism of the molecule*—that is, magnetism or polarity was said to be a property with which the molecule is *originally endowed*, and that magnetisation of a mass of iron consists in arranging the molecules all in one direction, not developing magnetism in them. This theory also assumed that neutrality was brought about by a haphazard arrangement of the molecules.

Prof. Hughes' experiments confirm the theory of the inherent polarity of the molecule, and prove that in neutrality the arrangement of the magnetised molecules is not haphazard, but perfectly symmetrical. He also proves that simple molecular rigidity explains all the effects of the retention of magnetism previously assigned to the mystical coercive force, and sums up his theory in these remarkable words :—

"From numerous researches I have gradually formed a theory of magnetism entirely based on experimental results, and these have led me to the following conclusions :—

"1. That each molecule of a piece of iron, steel, or other magnetic metal is a separate and independent magnet, having its two poles and distribution of magnetic polarity exactly the same as its total evident magnetism when noticed upon a steel bar-magnet.

"2. That each molecule, or its polarity, can be rotated in either direction upon its axis by torsion, stress, or by physical forces such as magnetism and electricity.

"3. That the inherent polarity or magnetism of each molecule is a constant quantity like gravity; that it can neither be augmented nor destroyed.

"4. That when we have external neutrality or no apparent magnetism the molecules or their polarities arrange themselves so as to satisfy their mutual attraction by the shortest path, and thus form a complete closed circuit of attraction.

"5. That when magnetism becomes evident, the molecules or their polarities have all rotated symmetrically in a given direction, producing a north pole if rotated in that direction as regards the piece of steel, or a south pole if rotated in the opposite direction. Also, that in evident magnetism, we have still a symmetrical arrangement, but one whose circles of attraction are not completed except through an external armature joining both poles.

"6. That we have permanent magnetism when the molecular rigidity, as in tempered steel, retains them in a given direction, and transient magnetism whenever the molecules rotate in comparative freedom, as in soft iron."

While referring the reader to the original Paper we may mention therefrom a few prominent points and experiments.

The rotation of the molecules under the action of an inducing field is rendered evident by taking a rod of soft iron and bending up the ends, by which a twist can be imparted to the rod while held vertically. The moment this is done the rod becomes magnetic, the field of the earth (assisted by the

molecular freedom imparted by twisting) causing all the molecules to be attracted and rotated in one direction. The lower end of the rod will be found to be north. On reversing the rod and applying a twist the molecules will rotate right round, and the external polarity will be reversed, the north pole being at the end which is now the lower. With a strip of soft Swedish iron, 1ft. long by $4\frac{1}{2}$ centimetres wide and $\frac{1}{2}$ millimetre thick, the writer obtained by twisting while held vertically a strength of pole equal to 7·3 C.-G.-S. units, as measured on the magnetometer. A mechanical model exhibited by Prof. Hughes to illustrate the rotation of molecules

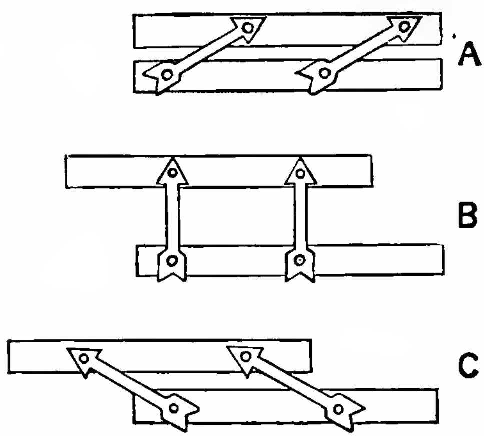


FIG. 98.

is shown in Fig. 98. It is on the principle of the parallel ruler, and is shown in the three positions in which A represents the direction of the molecules when a mass of iron is magnetised one way, C their direction when the polarity is reversed, and B the intermediate stage of transversal polarity. In Fig. 99, the molecules are shown as they exist in polarised iron or steel. The transversal polarity shown at *n s* has the external appearance of neutrality as far as the ends of the bar are concerned, but while there are external poles at all, although at the side,

there is no neutrality. We have only perfect neutrality when all the mutual attractions of the molecules are satisfied *amongst themselves* in the interior of the mass, and no external polarity is evident.

The inherent polarity of the molecules may be observed by drawing a rod of soft iron over the pole of a permanent magnet. This powerfully magnetises the rod, and it retains sufficient magnetism when withdrawn to strongly deflect a directing needle. A few twists of the rod then completely discharges its magnetism. If the operation of magnetising, withdrawing, and discharging were kept on we should expect to weaken the strength of the permanent magnet, and eventually draw all the magnetism out of it. But we do no such thing. The molecules are simply rotated each time, and the

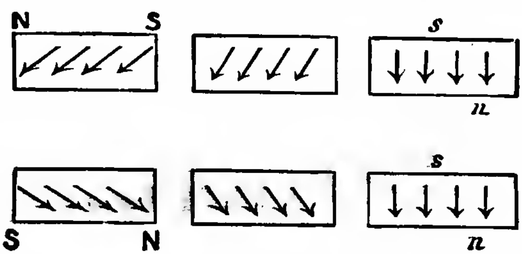


Fig. 99.

mechanical energy spent by the experimenter is proportional to the molecular rigidity of the metal.

Prof. Hughes has shown that by magnetising a rod in a given direction, and then slowly demagnetising it, and again magnetising it in the opposite direction, that there is no haphazard arrangement; every change takes place with perfect regularity and symmetry. In Fig. 100, at B and C is shown the neutral arrangement of the molecules, and at A a circular neutral arrangement effected by passing a current of electricity longitudinally through the rod.

To show the effect of torsion on the rotation of molecules, Prof. Hughes took a strand composed of about ten untempered

cast-steel drill wires, each 1 millimetre in diameter, and about 1 foot long, of the kind in use by watchmakers, and fastened handles at the ends of the strand, as shown (Fig. 101). Holding one handle fast, he imparted two entire right-handed twists to the strand with the other, and strongly magnetised it on the north pole of a magnet while under this torsion. He then released the two right-handed twists, and gave it two entire twists to the left, and oppositely magnetised it on the south pole while under the torsion. On releasing the torsion the strand possessed the remarkable property of a double polarity—that is, either end could be made north at pleasure by simply twisting it one way or the other. In the figure is shown the

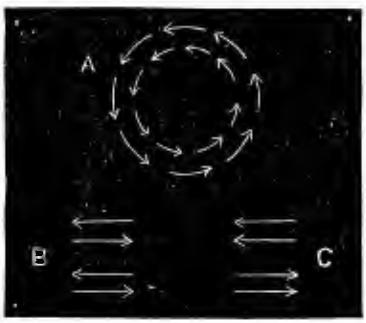


Fig. 100.

simple bell apparatus constructed by Prof. Hughes to demonstrate the marvellous transformation of one kind of mechanical movement to another through the medium of molecular motion. The magnetised armature *n s* is held in its position by the directive force of the magnet beneath it, and when the doubly-polarised strand is held near to the armature and twisted consecutively in alternate directions, the pole of the strand near to the armature changes in name for every alternate twist and produces attraction and repulsion of the armature, which therefore strikes alternately the rims of the glasses, producing a clear note for every twist given. The notes could be repeated in this way some six times per second.

In the same Paper Prof. Hughes notices a peculiar property of magnetism, viz., that besides the fact that the molecules of iron can be rotated in any direction when mechanical force is applied in the shape of blows, stress, or strain, it is also possible to rotate the molecules through a small arc under the influence of a very weak magnetic field, without the application of any mechanical force. The molecules appear to be surrounded by

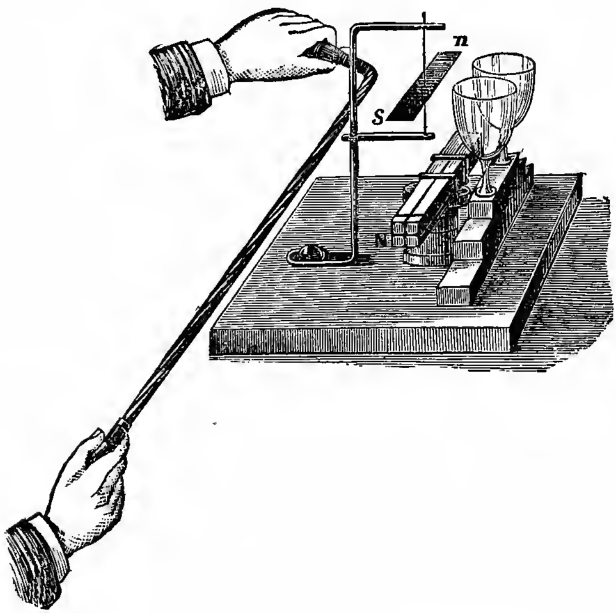


Fig. 101.

an elastic medium which permits of their free rotation through a small distance or arc under very feeble directing fields, and what is more remarkable is that this limiting distance of free motion, while it cannot be augmented, can easily be shifted so that the axes of free motion of the molecules are turned in any direction relatively to the bar. For instance, in many samples of iron this path of free motion may be observed at

once. The writer has taken a rod of iron 1ft. long and ½in. square, and found, when held vertically, strong north polarity at the lower and south at the upper end. On reversing the rod the same polarity has been observed, *i.e.*, north at the lower end. Hence, without the slightest mechanical shaking of the molecules, the latter have rotated under the weak directing field of the earth. If now a few blows be given the rod while held vertically it will be found that the axes of free motion of the molecules have been shifted, for the rod will now exhibit strong polarity one way and neutrality when turned the other way. The limit of free rotation has been found to have a distinct value for each class of iron. The extreme rapidity and sensitiveness of action of the telephone is probably due to the freeness of rotation of the molecules within this critical limit.

Section II.—Electro-Magnetic Fields.

127. **Preliminary.**—The telegraph systems of Morse and Wheatstone, with their relay and translating apparatus, electric bells and telephone exchange annunciators, systems for the synchronising of clocks, and a large proportion of the industrial applications of electricity, depend for their action upon that great discovery made by Arago and Davy nearly seventy years ago, viz., that pieces of iron, steel, and other magnetic metals become magnetised when an electric current is passed through a coil of insulated wire surrounding them. A magnet so formed is termed an *electro-magnet.* Incidentally here we may remark that metals which are said to be *magnetic* are not necessarily magnetised, the terms magnetic and non-magnetic indicating simply whether they are or are not *capable of being magnetised.* The two important principles of the electro-magnet may be thus stated :—

1. Reversal of the direction of current round the iron causes a reversal of its magnetic polarity.

2. Cessation of the current causes almost complete disappearance of magnetic polarity in the case of soft annealed iron.

To this might be added the important aid now available for the manufacture of powerful steel magnets magnetised electromagnetically with the aid of currents obtainable from dynamos and accumulators.

Considering the first principle—viz., that of reversal—it is found experimentally that if the current circulates in a clockwise direction round the iron core, the end of the same nearest to the observer is a south pole, and *vice versâ*. This will be seen to be the case in the two Figs. 102 and 103, although the direction in which the wire is wound on is different in the two cases.

The winding of the coil or helix in Fig. 102 is called right-

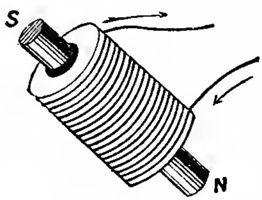


Fig. 102.

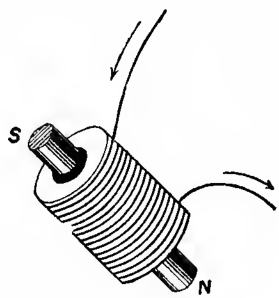


Fig. 103.

handed, the wire being wound on in a clockwise direction; while that in Fig. 103 is left-handed; and it will be seen that, in order to produce the same magnetic polarity in each, the current must be passed through in opposite directions in each as regards the ends of the coils. The current then circulates round each core the same way, producing the same poles in each. Reversing the current in either of these reverses the polarity.

An iron-filing diagram, taken with the electro-magnet of an ordinary telegraph "sounder," is shown in Fig. 104, a current being passed through the coils. The current was adjusted till the electro-magnet just supported the weight of the arma-

ture. Here we have precisely the same delineation of field as in the previous cases of permanent magnets. The two upright cores of soft iron are screwed to the horizontal iron keeper or "yoke" (Y).

The magnetic circuit is indicated by the dotted lines through the two cores and the yoke in Fig. 105, where it will be seen that this circuit completes itself through the air between the two poles by the curved lines of force, one typical line being shown in the figure. The direction assumed for these lines has already been explained (para. 104), and is marked by arrow heads in the figure. Now, when there is an iron "armature" in proximity to the poles of the electro-magnet, most of the lines of force are deflected, or turned from their original direction, and pass through the armature (Fig. 106). Noticing the direction in which these lines pass through the interior of the armature it will be clear (para. 104) that the latter becomes magnetised, its poles, *s n*, being as shown. Attraction must, therefore follow, and the armature is drawn towards the poles, N S, against the force of the counteracting spring. Every time the electrical circuit is completed by means of a "key" (*see* Fig. 34) this attraction occurs, and ceases when the cir-

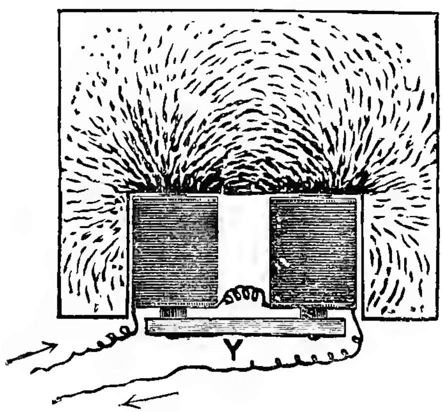


FIG. 104.

cuit is broken, by means of which, with the Morse code of signals made up of dots and dashes, signals can be sent from one end of a line to the other, and can be read by sound, or by the ink-writer, as the case may be.

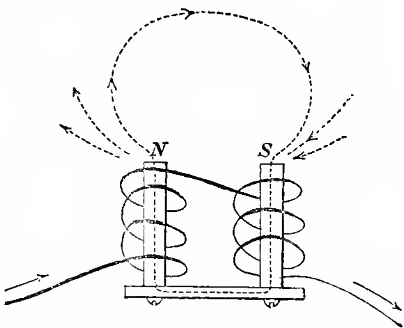


FIG. 105.

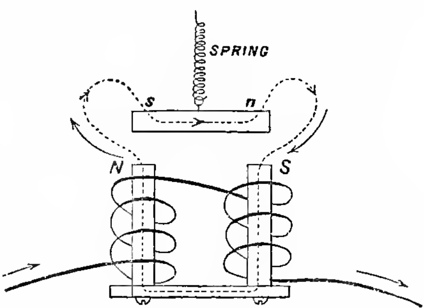


FIG. 106.

If we take an iron-filing diagram of the field when the armature is held away from the poles, or placed far enough away not to yield to the attraction (Fig. 107), the deflection of the lines of force through the armature is made evident; the position of the armature being seen by the absence of any

filings, since the latter only take up the position of the curves of force through the air. We have here between the armature and the poles of the electro-magnet what is termed "magnetic induction"—that is, the magnetism of the armature is "induced" in it by the action of the poles of the electro-magnet; or we may look at it another way, viz., the armature is placed in a magnetic field, and immediately becomes a magnet itself by "induction" from the field.

It will be seen by Figs. 105 and 106 that the current must pass round the two cores of the electro-magnet in the opposite direction to produce opposite poles at the upper end of the

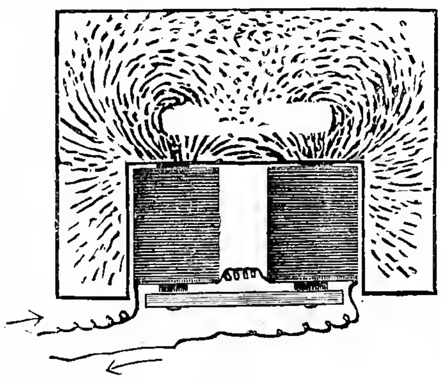


FIG. 107.

cores. The winding for this is easy to remember by considering the two cores as put end to end where they join the yoke, so as to make one bar; the winding is then a simple helix wound continuously along the bar, in order to produce opposite poles at the extremities.

In the rapid transmission of signals it is very important to overcome, as far as possible, the *time* taken by the electro-magnet to become magnetised and demagnetised. To overcome this retarding effect of the coils, which is a property depending on their self-induction, the two coils are wound in multiple arc (paras. 41 and 61), as shown in Fig. 108, for the

fast-speed Wheatstone receivers. More will be said upon this point when self-induction is considered.

If the cores were not connected together by the yoke, the poles of the electro-magnet would be much feebler, because there would then be two air gaps to be bridged over by the lines of force instead of one. Further, if a complete circuit of iron were formed by bridging over both upper and lower pairs of core ends by yokes, there would be many more lines of force created through the interiors of the cores, but there would be practically no external field. An external field is, however, required in the case of a "sounder" to cause attraction of its armature, and therefore the only air gap left is that immediately beneath it. The rule is, let there be a magnetic circuit

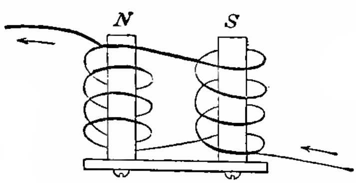


FIG. 108.

of iron everywhere except in the particular position where external magnetic action is required.

128. Magnetising Force.—The field of force set up in the interior of a coil of wire through which a current is flowing is found both by experiment and theoretical deductions to be proportional to the product of the strength of current and the number of turns of wire. If an iron bar be wrapped round with a given number of turns of wire, and a current of known strength passed through the wire, we have a definite field of force produced by the current in the coil which is capable of "exciting" or magnetising the iron. Hence we speak of the *exciting power* or *magnetising force* of the current, and in practice express it in *ampere-turns*. So long as the product of the number of amperes and number of turns is constant for a

given volume of wire, the magnetising force is constant whatever individual values the current or number of turns may have. This is the same thing as saying that the current must vary inversely as the number of turns for the same magnetising force. For example, if we have one electro-magnet, A, wound with 50 turns of thick wire, and another, B, wound with 1,000 turns of fine wire, the volume or weight of wire being the same in the two coils, the current strengths in the coils must be in the proportion of 1,000 to 50, or as 20 : 1 respectively, in order to produce the same magnetising force on the iron in each. It is sometimes more convenient to express the number of turns or length of wire in terms of the resistance. We have already seen (para. 63) that the length of wire wound on a coil of fixed volume or weight varies as the square root of the resistance; therefore, for a given magnetising force the product of the current and the square root of the resistance of the coil must be constant; or, in other words, the resistance must be inversely proportional to the square of the current. Hence, in the above example, the resistances of the two coils A and B are respectively as $1 : (20)^2$, or as $1 : 400$. Further, since the volume is constant, the cross-sectional area of the wire multiplied by the number of turns is constant, and therefore the sectional area of the wire on each coil is inversely proportional to the number of turns on each, and the diameter of the wires are inversely proportional to the square root of the number of turns. Hence, in the above example, the diameters of the wires on coils A and B are in the ratio of $\sqrt{20} : 1$ respectively. This is without taking into account space taken up by thickness of insulation. Similarly, the length of wire on each varies as the number of turns—viz., as 1 : 20 respectively.

We shall consider further on the C.-G.-S. unit of magnetising force.

129. Variation in Field due to more or less complete Magnetic Circuit.—Now let us measure with the magnetometer the strength of the external magnetic field when the magnetising force is constant and the amount of iron in the magnetic circuit

is varied. For this experiment two bobbins were taken (Fig. 109), each 20 centimetres long, and wound with No. 30 copper wire, to a resistance of $156 \cdot 5\omega$. This represented, therefore, about 2,180ft. of wire wound on each (No. 30 measures 13·95ft. to the ohm at 60° F.). Also, the mean diameter of each coil being 4·25 centimetres, the mean length per turn of wire was $4 \cdot 25\pi = 4 \cdot 25 \times 3 \cdot 14 = 13 \cdot 3$ centimetres $= \frac{13 \cdot 3}{30 \cdot 48}$ft. Hence the number of turns on each bobbin was

$$\frac{2180 \times 30 \cdot 48}{13 \cdot 3} = 5{,}000 \text{ turns approximately.}$$

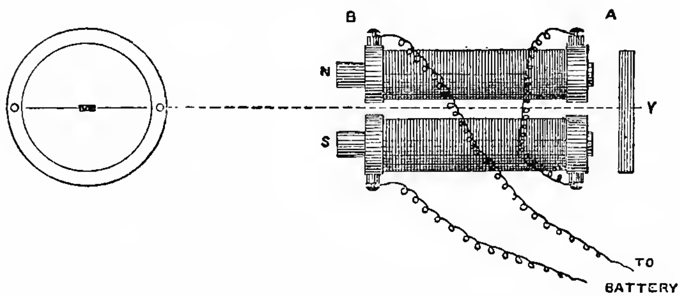


Fig. 109.

The two bobbins of wire were then connected in series to produce opposite poles at N S when the battery was connected. The cylindrical iron cores were 23·2 centimetres long and 2·5 centimetres diameter, and the two keepers or yokes, one of which, Y, is shown, were each 11 centimetres long and 2·75 square centimetres in sectional area. The parallel axes of the bobbins were placed 5·5 centimetres apart, and a potential difference of 42 volts connected to the terminals, maintaining, therefore, a current of

$$\frac{42}{156 \cdot 5 \times 2} = \cdot 134 \text{ ampere}$$

through the two coils, and a constant magnetising force on each coil of $\cdot 134 \times 5000 = 670$ ampere-turns.

The results obtained from the magnetometer readings are tabulated :—

—	Magnetometer deflection.	Magnetic Moment. C.-G.-S. units	Strength of Poles. C.-G.-S. units
Coils without iron cores ...	2	—	—
Iron cores inserted............	100	8,508	1,547
Yoke added at end A.........	440	38,030	6,915
Yoke added at end B.........	180	—	—
Last yoke removed and larger one substituted	0	—	—

The moments were determined in the manner already described (para. 125, 1st method), the perpendicular distance between the needle and the polar line of the electromagnet being 36·3 centimetres, and the deflection due to the coils alone being subtracted from each reading. The strength of pole was found by dividing the moment by the distance between the pole centres, 5·5 centimetres. The conclusions to be drawn from the above may be stated thus :—

1. A magnetic field is set up by the current in the coils alone.

(The deflection of two divisions at the distance of the coil from the magnetometer corresponds to a pole strength of about 30 C.-G.-S. units.)

2. The field is enormously increased by the insertion of iron cores.

Lines of force are evidently *created* by the presence of the iron in the field, since the magnetising force remains constant.

3. The field is still further increased by bridging across one air space by an iron yoke. Lines of force are again created, both through the interior of the iron and the remaining external air space, by making a more continuous iron circuit. We should, therefore, conclude that,

4. On completely closing the magnetic circuit by adding the second yoke at B, the field in the mass of the iron is still further increased, but that this tends to short-circuit the

external field through the air. The magnetometer, however, being still deflected considerably, we should conclude that

5. The mass of iron is not sufficiently large to carry the whole of the field so created, and therefore there is some leakage of lines of force through the air, which affect the magnetometer. This is borne out by adding a larger mass of iron ($15{\cdot}2 \times 5 \times 1{\cdot}5$ centimetres) in place of the small yoke at B, which effectually short-circuits the field—that is, causes the flow of lines of force to be entirely within the iron mass.

130. Intensity of Magnetisation.—An important branch of research tending to further the improvement of electrical apparatus and machines as well as to offer one of the most interesting fields for experimental investigation is that in which the relation between magnetising force and the resulting magnetism in different metals is studied. We shall refer later to some of the work which has been accomplished in this direction, but must first concern ourselves with the necessary fundamental bases on which comparisons can be effected in the degree or extent or quantity of magnetism acquired by masses of iron or steel, differing in dimensions and shape, when subjected to the action of magnetic fields of given intensity.

The two fundamental bases of comparison may be stated as:

1. The magnetic moment acquired per unit volume of metal, and termed the "Intensity of Magnetisation" (denoted by the letter I).

2. The number of lines of force per square centimetre of sectional area, forced through the mass of the metal, and called the "Magnetic Induction" (denoted by the letter B).

These two quantities were written in old English letters $\mathfrak{I}$ and $\mathfrak{B}$ by Prof. Clerk Maxwell, and the student will find them frequently written so in writings on the subject.

We shall now consider the first of these means of comparison. The conception of the idea of "intensity of magnetisation" is essentially one in which the strength of pole or amount of "free magnetism" is thought of. That is to say, the extent of the magnetism in the mass is expressed or thought of with

regard to the force of attraction or repulsion that can be exerted by its poles externally. The "action at a distance," or force exerted by a given pole strength, has been discussed already (para. 108), and it can be easily understood that we may have a number of bars of iron, of very different sizes, all of which exert the same external force, *i.e.*, have equal pole strengths, but in which the magnetism is distributed in very different proportions; or again, magnetised bars of similar size may have very different pole strengths. To effect comparisons, then, between the densities of magnetism—if one may so use the term—acquired by different bars, the pole strength is regarded as so much "free magnetism" distributed uniformly over the ends (shaded portions Fig. 110), and a comparison can then be made by finding for each bar what amount of free magnetism, or pole strength, exists at its ends *per square centimetre of sectional area*. This is termed the "intensity of magnetisation," or sometimes simply "magnetisation," and is obviously the same as magnetic moment per cubic centimetre. Take a bar 10 centimetres long and 2 square centimetres sectional area. If its pole strength is 1,000 C.-G.-S. units, the intensity of magnetisation is found by

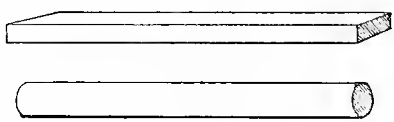


FIG. 110.

$$\frac{\text{pole strength}}{\text{sectional area}} = \frac{1000}{2} = 500 \text{ C.-G.-S. units};$$

that is, for every square centimetre of end area there are 500 units of pole strength. It is evident that if we multiply both numerator and denominator of the above by the length, we have

$$\frac{\text{magnetic moment}}{\text{volume}} = \frac{10000}{20} = 500 \text{ C.-G.-S. units,}$$

which is, therefore, the same thing, and has the same value, and gives us the usual definition for the intensity of magnetisation, viz., the magnetic moment per unit volume (per cubic centimetre on the C.-G.-S. system). Obviously this is the more generalising definition, as it covers irregularity of sectional area. As an example of the comparison of magnets of the same size but of different pole strengths by their intensities of magnetisation we may take the two magnets already mentioned (para. 125, examples 3 and 4), which were found to have pole strengths of 554 and 527 C.-G.-S. units respectively. They were flat bars, measuring at the end 2·8 centimetres by 1·3 centimetre, and therefore of $2{\cdot}8 \times 1{\cdot}3 = 3{\cdot}64$ square centimetres sectional area, uniform throughout. Their intensities of magnetisation were therefore

$$\frac{554}{3{\cdot}64} = 152 \text{ C.-G.-S. units}$$

and

$$\frac{527}{3{\cdot}64} = 145 \text{ C.-G.-S. units.}$$

The intensity of magnetisation reached when soft iron is subjected to powerful electro-magnetic fields may be very much higher than the above, even to values nine or ten times as high, although the above are average intensities ordinarily met with in new steel *permanent* magnets.

To further illustrate this method of comparison the following measurements taken with wrought and cast-iron bars are cited. A constant difference of potential of 42 volts was kept at the terminals of the magnetising coil (Fig. 111), which was one of those previously described (para. 129, Fig. 109), the current, and therefore the magnetising force, being twice as much as that previously given for the two coils together under the same potential difference, viz.,

$$670 \times 2 = 1340 \text{ ampere-turns, approximately.}$$

We shall show in the next section on the magnetic fields of coils without cores, or solenoids, that the magnetising force expressed in C.-G.-S. units (lines of force per square centimetre) at the central cross section of a long solenoid of length

l centimetres, and excited by a current C, in C.-G.-S. measure is equal to

$$\frac{4\pi C n}{l} \text{ C.-G.-S. units intensity of field,}$$

the length of the solenoid being great compared to its diameter, and forming a long narrow coil. We anticipate, however, for the purpose of showing the relation between ampere-turns and C.-G.-S. units of magnetising force. The C.-G.-S. unit of current is equal to 10 amperes, so that if x ampere-turns are equal to one C.-G.-S. unit we have

$$\frac{\text{ampere-turns}}{x} = \text{field in C.-G.-S. units,}$$

and therefore

$$\frac{10\,C n}{x} = \frac{4\pi C n}{l}$$

(C being in C.-G.-S. units on both sides of the equation),

whence

$$x = \frac{10\,l}{4\pi} = \frac{l}{1 \cdot 257},$$

and therefore

$$\text{field in C.-G.-S. units} = \frac{\text{ampere-turns}}{\text{length of coil (centimetres)}} \times 1 \cdot 257.$$

In the above example, therefore, which is a coil 18·5 centimetres in length, instead of 1,340 ampere-turns of magnetising force, we may put it

$$\frac{1340 \times 1 \cdot 257}{18 \cdot 5} = 91 \text{ C.-G.-S. units.}$$

The magnetising force is usually denoted by the letter H, or in Maxwell's notation by its old English form $\mathfrak{H}$.

It will be noticed that the electromagnet is placed, as previously described, for measuring the moment by the first method of Gauss (para. 125), the distance of its centre from that of the magnetometer needle being 50 centimetres. First of all, before putting any core in the coil, its own field with the current on was measured. This deflected the magnetometer 11 divisions, and therefore in the measurements of

the moments of the cores this deflection was deducted from the readings. Three cylindrical cores similar to C C, Fig. 111, but of different dimensions, were tried, with the following results :—

Cores.	Length in cms. (l)	Diam. in cms. (d)	Magneto-meter deflection.	Moment in C.-G.-S. units. (M)	Intensity of Magnetisation. (I)
I. Wrought Iron	30·7	2·3	420	106,300	833
II. Ditto	26·9	1·9	280	67,210	881
III. Cast Iron ...	30·8	2·3	280	69,350	541

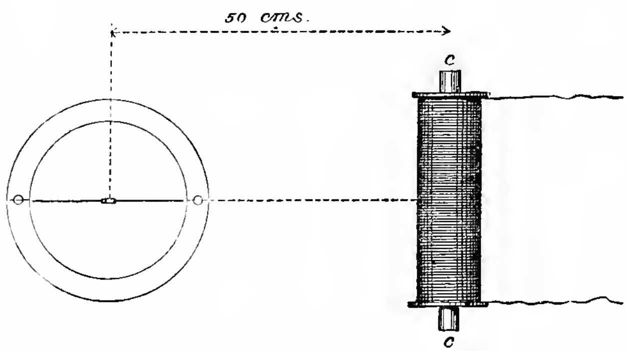


Fig. 111.

Here the dimensions of core differ, and therefore the magnetometer readings and calculated moments offer no basis of comparison as to the density of free magnetism acquired by each. This is seen notably in the last two cores, where the deflections are equal, and the moments not very widely different ; but the intensity of magnetisation, which forms the basis of comparison, shows one little more than half the other.

It may be of assistance to the student to see one of these examples worked out. Take the first :—

$$\tan\alpha = 4{\cdot}20 - {\cdot}11 = 4{\cdot}09$$

$$\text{earth's field (H)} = {\cdot}18$$

$$r^3 = (50^2 + 15{\cdot}35^2)^{\frac{3}{2}}$$

and (*see* para. 125)

$$\text{moment} = r^3\,\text{H}\tan\alpha = 106{,}300 \text{ C.-G.-S. units,}$$

$$\text{sectional area} = \frac{\pi\,d^2}{4} = \frac{3{\cdot}1416 \times 2{\cdot}3^2}{4} = 4{\cdot}147 \text{ square cms.}$$

$$\text{volume} = \text{area} \times \text{length} = 4{\cdot}147 \times 30{\cdot}7 = 127{\cdot}3 \text{ cub. cms.,}$$

and intensity of magnetisation

$$\text{I} = \frac{\text{moment}}{\text{volume}} = \frac{106{,}300}{127{\cdot}3} = 833 \text{ C.-G.-S. units.}$$

The low magnetisations obtained for the above-mentioned magnetising force are most probably due to the estimation of the number of turns of wire in the bobbin (para. 129) being too high. It was not wound specially for the purpose, and the exact length of wire and number of turns were not known. Further, no mechanical stress or blows were given to the iron while under the magnetising field, which would have increased the magnetisation acquired.

When the core is not of uniform sectional area, if its volume cannot readily be computed by direct measurement it may be determined for a large core by immersing it in water, and measuring the volume of water thereby displaced. Or, again, for a small core, such as a piece of iron wire, it may be found by weighing in air and then in water, the ratio of the weight in air to the loss of weight in water being the specific gravity, and, in the C.-G.-S. system the weight of 1 cubic centimetre of water at 4°C. being 1 gramme, we have

$$\text{volume in cubic centimetres} = \frac{\text{weight (in air) in grammes}}{\text{specific gravity}}.$$

Substituting for specific gravity, we have

$$\text{volume in cubic centimetres} = \text{loss of weight in water (in grammes).}$$

The determination of the intensity of magnetisation is not confined to methods like the above, which have for their *modus operandi* the measurement of the external field of force of the magnetised bar, but may be calculated from the value of the magnetic induction B when that has been determined by a different method (presently to be detailed) and when the magnetising force can be accurately computed.

The importance of the subject of electro-magnetism as regards our knowledge of iron and steel in various stages of magnetisation, may be conceived when it is borne in mind that most of the men whose names are most honoured amongst us, as much in some for their high scientific attainments as in others for their practical experience, insight, and intuition, have given to this field of research their careful thought and investigation, and that what we know and make use of to-day is the result of incessant experimental investigation extending over, in many cases, years of patient toil. It will be the aim of the writer to point out some of the most salient results arrived at, and to endeavour to render clear to the student the terms used in connection with this subject.

131. **Apparatus for Measuring the Relation between Magnetising Force and Intensity of Magnetisation.**—We shall now consider some of the practical details of a simple test to measure the changes in the intensity of magnetisation acquired by an iron or steel rod, when the current in the coil which surrounds the rod is gradually increased. The practical example given below of a test of this kind will serve as a guide, although the effects may be observed equally well with iron wires of much smaller diameter and currents of less strength than those employed below. The apparatus is connected up as in Fig. 112, where the magnetometer M and distance table are shown in plan, the current meter G and adjustable resistance R in elevation, and the battery, in this case a secondary battery of 21 cells, is shown conventionally. The galvanometer is placed some five or six feet away from the magnetising coil C, and is connected to the rest of the

circuit by well insulated and twisted leading wires (para. 67), in order that its needle may not be affected by any magnetic field but that set up by its own coil. A brass tube of about $\frac{1}{2}$ centimetre bore and a little over a foot long was provided with cheeks at the ends, fixed so that the distance between them was exactly 1ft. The tube was then

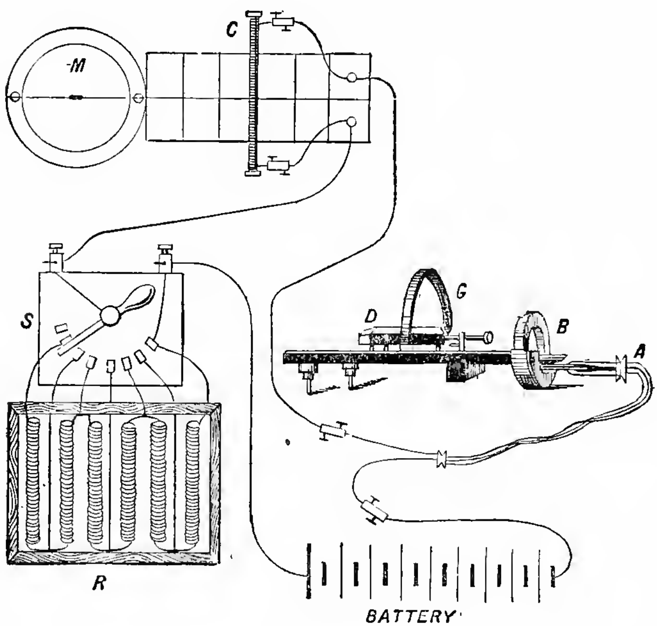


FIG. 112.

wrapped round evenly with No. 16 double cotton-covered copper wire, and the ends firmly secured by twine. This gave a magnetising coil (C in the figure) 1ft. long and of 135 turns, which was then placed on the table at 20 centimetres distance from the magnetometer needle in the position shown, and held there firmly by a wooden clamp.

The resistance R is made with German silver wire spirals connected in series and fastened to a wooden frame. German silver (an alloy of 2 parts copper, 1 part zinc, and 1 part nickel) is chosen because less length of wire of a given thickness is required than if copper or iron were used, owing to the higher specific resistance (para. 29) of this alloy, which is some 12 or 13 times that of copper. Less space is therefore taken up by the coils. The bights between each spiral are soldered to wires running to contact stops on the switch S. When the switch lever rests on the extreme left hand contact the circuit is disconnected, when on the second stop (as in figure) the circuit is closed, and a current will flow from the cells through all the resistance coils, the magnetising coil, and galvanometer. The current may then be increased step by step by moving the lever to the right. It is advisable to make the spirals of very low resistance on the low-resistance side (right side) of the switch, say of a quarter and half ohm, owing to the fact that when the rest of the circuit is of low resistance, very slight changes in the adjustable resistance produce great variations in the current strength. The spirals near this end of the switch should also be of thicker wire, say No. 14 or 16 B.W.G., as they have to carry the heaviest currents, while the rest may be of No. 18, and wound in spirals of 1, 2, and 5 ohms. The best arrangement, where great variation in resistance is required, is to have two separate switches; for example, suppose the total variation in resistance required was about 20 ohms. If one switch (A) of seven contacts were connected to six coils, in the manner shown in the figure, three coils being of five ohms each, and three of a quarter ohm each, and another switch (B) of five contacts connected to four coils, each of one ohm, the total resistance would be $19\frac{3}{4}$, and it could be reduced by one ohm at a time down to four ohms, and below four by a quarter ohm at a time by working the two switches. Supposing all the resistance in to start, B would be reduced four ohms by one ohm at a time first, and then its switch put back so as to add the four ohms again; then A would be moved one stop, reducing five ohms, and then again B one ohm

at a time, and so on. In a test of this kind, where we are measuring the magnetisation of iron by gradual increments of current, care must be taken that if the resistance is reduced too much by mistake in one step, the reading on the magnetometer must be taken with *this* resistance in ; it is no use to increase it up to what was intended at first, as the increased magnetisation of the iron due to the larger current is not lost entirely on decreasing the current. The actual resistance in ohms is not required to be known in this test, so that the lower variations in resistance may be obtained very well by two similar metallic plates of, say, a foot square, immersed in slightly acidulated water, whose distance apart can be varied to a maximum distance of about two feet.

We must now turn our attention to the galvanometer. At the outset it will be noticed that in a test of this kind we are varying the strength of the current within fairly wide limits. In the test presently to be detailed, taken with the above apparatus the current was a little over half an ampere to start with, and was gradually increased to nearly 40 amperes. Now, with wide variation in current the galvanometer must have different degrees of sensitiveness in order to allow of reading the deflections with the least chance of error. Suppose a large current through the instrument gives a conveniently large deflection, then if the instrument remained in the same state of sensitiveness, as the current was reduced, the deflections would get smaller and smaller till the very smallness of the deflection would prevent accurate readings being taken, the movement of the needle being very slight for given changes in the current either at very low or high deflections from zero. Now, if the sensitiveness of the instrument is increased n times the deflections will be n times as large, and therefore the feebler currents can be read with ease and accuracy. The converse holds in the case before us. We are increasing the current strengths, and therefore must *decrease* the sensitiveness of the galvanometer as we proceed. We may do this in four ways, viz. :—

1. Shunting the galvanometer at intervals with shunts of gradually decreasing resistance.

2. Increasing the magnetic controlling field by bringing the controlling magnet nearer the needle.

3. By using a galvanometer wound with four or five separate coils, commencing with the fine wire coil of many turns, and changing at intervals on to the thicker wire coils of decreasing number of turns.

4. By gradually moving the needle and scale further from the coil.

As we wish to measure the currents accurately, and are working from a half to 40 amperes, the first method would be impracticable. When the current increased towards its larger values the shunts on the galvanometer would require to be so low in resistance that they would be difficult of accurate measurement, to say nothing of providing them of thick wire or stout copper ribbon, so that their resistance should not alter by heat; and to calculate the total current from the deflection of a shunted galvanometer, we must, of course, know accurately the resistance of the shunt (para. 95). As regards the second method of shifting the controlling magnet, the readings might first be taken without the magnet, and when beyond the limit of accurate reading, say 50 degrees, the magnet would be placed at a vertical height over the needle, such that the deflection of 50 was halved (or if a tangent galvanometer, a position found for the magnet, such that the deflection was the angle whose tangent was half the tangent of 50 degrees). The magnet would be kept in this position until 50 degrees deflection was reached again, and then lowered to a position in which the deflection of 50 was halved, and so on. For each re-adjustment, we should know that the same strength of current gave half the deflection, and therefore we should multiply the deflections (or their tangents) by 2, 4, 8, 16, 32, &c., for each consecutive change. The galvanometer would have to be wound, however, with wire thick enough to carry the strongest current without dangerous heating, and therefore could not hold many turns.

The third method requires a specially-wound galvanometer. If such an instrument is available it is very convenient to use, since the constants (para. 100) may all be determined beforehand, once for all. The resistance of the coils is not taken into account, the variation in sensitiveness, as regards current, being effected only by the different number of turns. The reason why the coils of fewer turns are made of thick wire is that they may carry the large currents sent through them without heating sufficiently to cause injury to the insulation.

The last method can best be explained by describing Sir William Thomson's graded galvanometer, the sensitiveness of which can be varied in this way, and, in fact, was the instrument used for the test we are now considering. We shall give in our next chapter a detailed description of galvanometers, and, although it may be considered as deviating widely from usual custom to digress from the main subject in hand in this manner, it is hoped that the reader will not find it altogether without interest to have instruments explained at the time they are used.

132. Sir William Thomson's Graded Galvanometer.—This instrument (represented in elevation at G, Fig. 112, and in plan in Fig. 113) has a wide range of sensitiveness, effected by varying the distance between the coil and needle. The coil, shown at B, is fixed in position and wound with six turns of copper strip 1·2 centimetre wide and 1·5 millimetre thick, each turn being insulated from its neighbour by winding on, side by side with the copper strip, a ribbon of asbestos paper. The two ends of the copper strip are brought out at T, side by side, being kept apart by a wood strip about $\frac{1}{4}$in. thick, gradually tapered down towards the end, to admit of sliding on the connecting clip A, from which two twisted leading wires connect the coil to the rest of the circuit. The advantage of this mode of connection is that the instrument may be introduced into or withdrawn from a circuit without causing temporary interruption. The quadrant-shaped box D, with glass cover, contains the pivoted system of needles and pointer, and is fitted with a mirror and tangentially divided scale. The instru-

ment being tangential, the deflections on the scale are proportional to the currents. There are four little magnets, two of which are seen in the figure; the other two are beneath these, and the whole are fixed in an aluminum frame which is prolonged to form a looped pointer. The needle-box may be moved to any position, the V groove cut in the platform guiding it so that the needle is always in a line with the

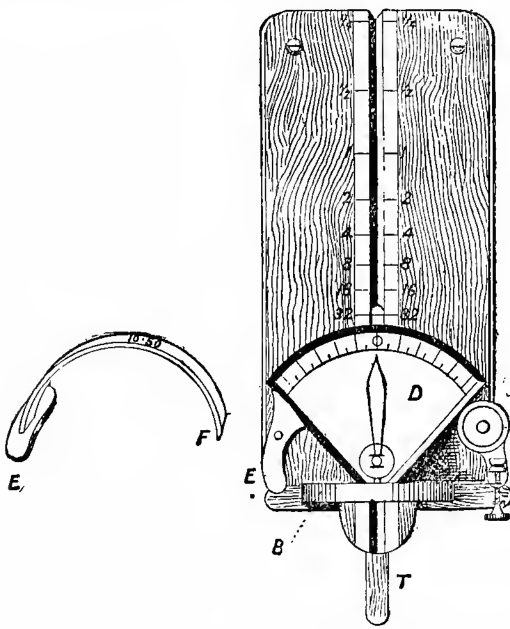


Fig. 113.

centre of the coil. The two ends E and F of the controlling magnet can be fitted into the side arms E and F of the needle-box, so producing an additional controlling field on the needle in the direction of the meridian, the value of which and the date when measured are usually painted on the magnet. This value cannot, however, be depended on in practice, unless the

greatest care is taken to keep the magnet from getting knocked or being placed within range of the influence of foreign magnetic fields, and it is therefore advisable to measure it before or after use. A pin on the underside of the magnet at E fits into a hole in the arm E, and the point of the magnet at F fits in the grooved wheel carried by the adjustable screw at the end of the arm F, by means of which screw the magnet may be accurately adjusted to the plane of the meridian. The arm F also carries a spirit level. Fixed to the outside of the quadrant, opposite the zero point of the scale, is a little prolongation with a horizontal cut in it, the use of which is to enable the observer to bring the needle-box exactly into a given position of sensitiveness by observing that the cut and the given mark on the platform coincide. In the figure this is placed at position 32. At position 1, without the controlling magnet, the needle is at such a distance from the coil that the current producing one division deflection is exactly equal numerically to the intensity of the earth's horizontal field H at the place where the instrument is used, which, in London, is ·18 C.-G.-S. units—that is,

$$\text{Current} = \cdot 18 \text{ ampere per division.}$$

Hence, for D divisions,

$$\text{Current} = D \cdot 18 \text{ amperes.}$$

Now, at positions 2, 4, 8, 16 and 32 the sensitiveness of the instrument is increased twice, four, eight, sixteen and thirty-two times respectively; that is, a current deflecting the needle, say, 10 divisions at position 1 will deflect it 20 divisions at position 2, 40 divisions at position 4, and so on. Or if we put currents through the instrument, to cause equal deflections when the needle-box is at positions 1, 2, and 4, these currents would be in the proportion of 4, 2 and 1 respectively. Hence the currents vary inversely as the position P of the needle-box on the platform for a constant deflection, and we may write—

$$\text{Current} = \cdot 18 \frac{D}{P} \text{ amperes.}$$

The field F produced at the centre of the needles by the controlling magnet, when this is used, must be added to that of the earth H, as the two fields are in the same direction. To find this field it is necessary to pass a current through the instrument, and note first the deflection without the magnet, then notice how many times the deflection is reduced, say n times, by adding the magnet. This is best done by having the needle-box much nearer the coil when the magnet is on; for example, a constant current is kept on, and at position $\frac{1}{2}$, without the magnet, the needle is deflected, say, 22·5 divisions; the current is then

$$\cdot 18 \frac{22\cdot 5}{\cdot 5} \text{ amperes.}$$

Now, at position 32, with the magnet on, the needle is deflected, say, 30 divisions; the current is then

$$\cdot 18 \frac{30n}{32} \text{ amperes,}$$

where n is the number of times the sensitiveness is reduced by adding the magnet. The current being the same, the above two expressions are equal, and

$$\frac{22\cdot 5}{\cdot 5} = \frac{30n}{32}, \qquad \text{whence } n = 48.$$

The value of the resultant controlling field is then 48 or n times H, and we have seen above that it is also equal to H + F, where F is the field due to the magnet; therefore

$$\begin{aligned} H + F &= n\,H \\ F &= H\,(n-1) \\ &= \cdot 18 \times 47 = 8\cdot 46 \text{ C.-G.-S. units.} \end{aligned}$$

It is the value of F thus measured which is usually painted on the magnet, together with the date when measured.

Now, it will be seen in the figure that the needle is not within the coil at position 32; there is still a position of higher sensitiveness, which does not usually fall in the same geometrical series as the previous grades. Its value in some

instruments is over 40. If we take it as 45, and suppose the needle-box placed there, without the controlling magnet, we should have

$$\frac{\cdot 18}{45} = \frac{1}{250} \text{ of an ampere per division.}$$

And, using the above magnet with the needle-box at position $\frac{1}{4}$, we should have

$$\frac{(\cdot 18 + 8\cdot 46)}{\cdot 25} = 34\cdot 5 \text{ amperes per division.}$$

Between these two extremes, which include a very wide range, any degree of sensitiveness may be obtained.

133. The Demagnetising Force set up by a Magnetised Rod.—A bar or rod which has been magnetised sets up by the action of its own poles a reverse magnetising force—that is, it tends to demagnetise itself. To study this phenomenon we must consider the quantity of lines of force issuing from a pole of strength m (the action of the complementary pole being considered negligible). At a distance of r centimetres from the given pole in *all directions* the magnetic field is equal to

$$\frac{m}{r^2} \text{ C.-G.-S. units}$$

(*see* paras. 108, 109, and 113)—that is to say, the above number of lines of force pierce every square centimetre of surface of a sphere of radius r centimetres whose centre is the pole m. But on the surface of such a sphere there are $4\,\pi\,r^2$ square centimetres area, through each of which, as we have seen, pass the above number of lines of force. Therefore, the total number of lines issuing from the pole m is equal to

$$4\,\pi\,r^2 \times \frac{m}{r^2} = 4\,\pi\,m \text{ lines.}$$

(The Greek letter π (*pi*) is always used as the ratio of the circumference of a circle to its diameter, which is $3\cdot 14159$.)

Now take an iron bar (Fig. 114) to which is applied a certain magnetising force, the *direction* of which is indicated by the straight arrows on each side of the bar. Suppose that the iron acquires m C.-G.-S. units strength of pole.

Lines of force will issue from the N pole and pass in curves through the air to the S pole (para. 104); but these are not all the lines issuing from the pole. They issue radially in all directions, and therefore some pass through the metal to the S pole. It is precisely these lines *within* the metal which

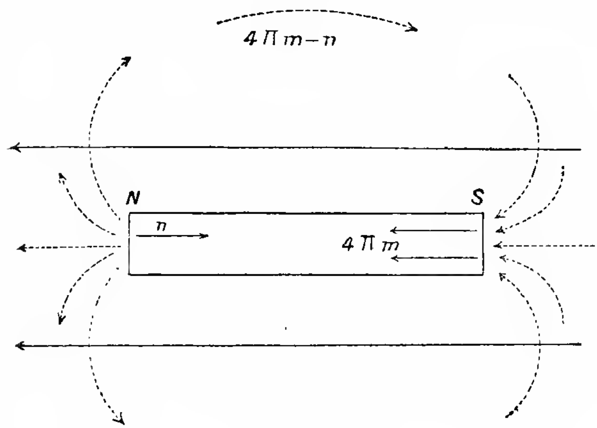


FIG. 114.

tend to demagnetise the bar. Say there are n demagnetising lines running from N to S through the metal; we shall then have $4\pi m - n$ lines issuing from the N pole into the air, $4\pi m$ being the total number. Both the lines through the air and the demagnetising lines through the metal form closed circuits, and therefore at the S pole the two return circuits of lines are in the same direction and run back through the metal from S to N. These are respectively $4\pi m - n$ and n. Adding them together, because they are in the same direction, the demagnetising lines are eliminated, and we have $4\pi m$ as the number of lines through the metal

in the same direction as the magnetising force applied to the bar. This phenomenon is all-important in magnetisation experiments. We wrap a bar of iron round with a certain number of turns of insulated wire, pass a known current through the wire, and from these data can calculate exactly what number of magnetising lines of force are produced by the coil, viz.,

$$\frac{4\,\pi\,C\,n}{l} \text{ lines per square centimetre}$$

(*see* para. 130). But as soon as the iron becomes magnetised, the action of its own poles causes n *demagnetising* lines to pass through the metal and weaken the above magnetising field. Now if the sectional area of the bar is a square centimetres, the demagnetising lines are

$$\frac{\mathrm{n}}{a} \text{ lines per square centimetre,}$$

and therefore the resultant, or actual magnetising force **H**, acting on the bar is

$$\frac{4\,\pi\,C\,n}{l} - \frac{\mathrm{n}}{a} \text{ lines per square centimetre.}$$

Without knowing the value of n, therefore, the magnetising force cannot be exactly estimated. Now, in experiments to determine the magnetic "susceptibility" and "permeability" of specimens of iron, it is essential to know the exact magnetising force applied, and this is practically accomplished by reducing the demagnetising force to such a small relative value that it can be neglected. Now, the demagnetising force is greater in proportion as the poles are stronger and nearer to each other, and in the case of bars we can only, with strict theoretical exactness, get rid of its effect by using rods of infinite length. Practically, however, by using rods whose length is many times their diameter, the demagnetising effect may be neglected. Prof. J. A. Ewing, of University College, Dundee, who is a high authority on the electro-magnetisation of iron, says in his Paper, entitled "Researches in Magnetism"

(*Phil. Trans.*, 1885) :—"My own observations show that it is only when the length of the rod (if of iron) is about 300 or 400 times its diameter that the effect of length becomes insensible."

The same authority also states that the demagnetising force due to the ends is unequal along the bar, being much greater at the ends. The demagnetising effect is entirely got rid of if the iron is employed in the form of a ring, since, in this form, there is a complete magnetic circuit, and therefore no poles. The intensity of magnetisation can be found indirectly this way from the value of the magnetic induction **B**, and the magnetising force **H** ; but what we are considering now is the *direct* measurement of the magnetisation by the action of the poles on the magnetometer.

We shall select two instances from a number of experiments to illustrate the difference between these conditions. Take, first a short, thick iron rod, in which there is considerable demagnetising effect. When the demagnetising action of the poles is got rid of by employing either very thin iron wires or iron rings, the condition is technically spoken of as that of "endlessness."

134. Electro-Magnetisation of Short Iron Rod.—The necessary apparatus having been explained, we shall detail the method of taking observations, to show the gradual magnetisation of a short iron core placed within a coil of wire through which definite currents are passed. Referring to the apparatus, as arranged in Fig. 112, the magnetometer and galvanometer must first be levelled and adjusted in position till their respective pointers coincide with the zero marks on the scales. If the controlling magnet is employed with the graded galvanometer described in para. 132, it must first be removed several feet away while the instrument is being levelled and adjusted to zero. The latter is effected by turning the instrument round until the pointer is at zero, and the levelling is effected by the two screws seen in the figures. The controlling magnet is then placed in position, and if not in the plane of the meridian will deflect the pointer a little away from zero.

To rectify this the adjustable screw on the arm F of the needle box is turned until the pointer is restored thereto. By a measurement, taken immediately before the readings, the controlling magnet used was found to reduce the sensitiveness twenty-one times (n). Its field at the needle was therefore (para. 132)

$$H\,(n-1) = \cdot 18 \times 20 = 3\cdot 6 \text{ C.-G.-S. units.}$$

The first thing to be done is to put two or three different strengths of current through the coil alone, and read the corresponding deflections on the two instruments. From these observations, when plotted on squared paper, it can be seen what must be deducted from the readings taken with coil and core together to give the true magnetic effect of the core. Having done this, the current is switched off and the core inserted into the coil.

The core used in this experiment was a cylindrical rod of unannealed iron, 10 centimetres long and 4·3 millimetres diameter. Its sectional area was therefore

$$\frac{\pi\,d^2}{4} = \cdot 7854 \times \cdot 43^2 = \cdot 1452 \text{ square centimetres,}$$

and its volume $= \cdot 1452 \times 10 = 1\cdot 452$ cubic centimetres.

The length of the core being only some 23 times its diameter, there was considerable demagnetising effect. The core being shorter than the coil, care was taken by measurement to push it into an exactly central position in the coil. The latter being held down by a clamp this operation could not shift its position.

Now it is important to notice at this point whether the magnetometer is deflected; if it is, the core has some traces of magnetism which should be removed before commencing the test. This may be done by heating it and then allowing it to cool while lying horizontally in an east and west direction; or a few light taps at the end while held in the same position will generally suffice.

The resistance switch is now turned so as to put the greatest amount of resistance in circuit, and the current is then put on.

Both magnetometer and galvanometer deflections must now be read as each successive reduction is made in the resistance, and the same jotted down in two vertical columns. Some of the readings are here appended, the galvanometer deflections being divided into columns corresponding to the position of the needle-box on the platform when the respective readings were taken.

Magnetometer Deflections.		Galvanometer Deflections.					Ampere-turns.
		4	8	16	32	Reduced to 32	
Coil	17	$35\frac{1}{4}$	—	—	—	282	4512
Coil and Core	2	—	—	—	5	5	80
	14	—	—	—	20	20	320
	38	—	—	$23\frac{1}{2}$	—	47	752
	72	—	—	45	—	90	1440
	102	—	$35\frac{1}{2}$	—	—	142	2272
	113	$26\frac{1}{4}$	—	—	—	210	3360
	126	33	—	—	—	264	4224
	131	$41\frac{3}{4}$	—	—	—	334	5344

It is convenient afterwards to reduce them all to their value at position 32. At this position, with the controlling magnet in use, the current is equal to

$$(\cdot 18 + 3\cdot 6)\frac{D}{32} = \frac{D}{8\cdot 47} \text{ amperes,}$$

where D is the deflection reduced to position 32, and 3·6 (as given above) is the field of the controlling magnet, which was used throughout. The number of turns on the magnetising coil being 135, we get the number of ampere-turns by multiplying the current by this number—that is,

$$\text{ampere-turns} = \frac{D}{8\cdot 47} \times 135 = 16\ D.$$

The deflections reduced to position 32 and multiplied by 16 give, therefore, the magnetising forces in ampere-turns, these values being worked out in the last column of the above table.

The curve in Fig. 115 is plotted on squared paper from the readings. The horizontal ordinate is divided off in divisions,

each representing 1,000 ampere-turns, and the above values marked off accordingly. On the vertical ordinate it is more convenient to plot the magnetometer deflections first, which are proportional to the field produced by coil and core together, as we have to subtract the effect of the coil before we can estimate the magnetisation of the core, and this is most readily done by plotting the deflections. This gives a straight line

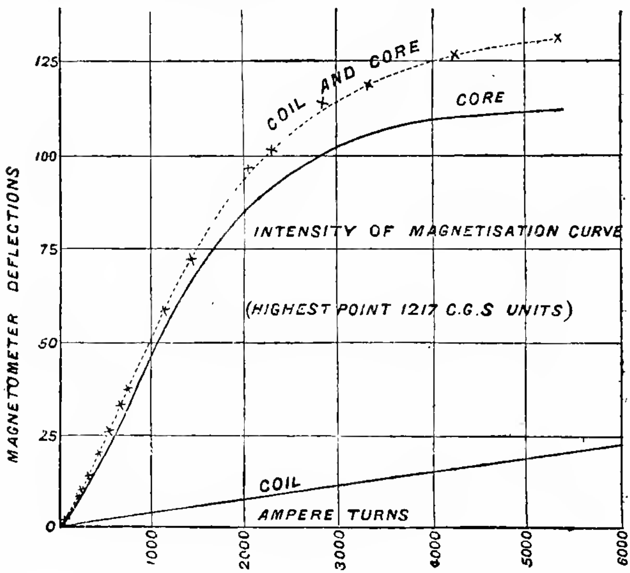


Fig. 115.

for the coil alone, and a regular curve (shown dotted) for the variation in external magnetic field of the coil and core together. Taking now a point on the horizontal ordinate for any given number of ampere-turns, the height of the vertical line from this point to the straight line of the coil must be subtracted from the height of the vertical line from the point to the dotted curve; this gives one point on a new curve which represents the effect of the core alone. This operation

having been repeated for several points on the base line, we get a corresponding number of points on the new curve, which may then be connected by a line, giving the curve shown. By inspection of this curve we can see what deflection the magnetometer would have indicated had it been acted upon by the core alone. Calling this angle of deflection α, the tangent of which, as before explained, is equal to the deflection divided by 100, we may find the intensity of magnetisation of the core corresponding to any given point in the curve by

$$\mathsf{I} = \frac{\text{moment}}{\text{volume}} = \frac{r^3 \text{ H} \tan \alpha}{1 \cdot 452} \text{ C.-G.-S. units.}$$

Now $$r^3 \text{ H} = (20^2 + 5^2)^{\frac{3}{2}} \times \cdot 18 = 1577,$$

and $$\frac{1577}{1 \cdot 452} = 1086.$$

Therefore, $$\mathsf{I} = 1086 \tan \alpha.$$

Take the highest point in the curve; the deflection is 112, and therefore

$$\mathsf{I} = 1086 \times 1 \cdot 12 = 1217 \text{ C.-G.-S. units.}$$

Values of I can be worked out in this way for any other point on the curve. It should be noticed, however, that the shape of the magnetisation curve is the same whether we plot strength of pole, moment, intensity of magnetisation, or deflections on magnetometer due to core, since all these quantities are proportional to one another. Similarly, the curve will be the same whether the magnetising force is plotted in ampere-turns or C.-G.-S. units.

The foregoing example is detailed with the view of furnishing the student with material sufficient to guide him in observations and calculations of his own. Experiments made on however small a scale, the results of which are reduced to known units of measurement, are the best means of acquiring a clear conception of terms and their relative values. Near the higher part of the curve, although the readings were taken

as rapidly as possible, the current perceptibly warmed the coil and core, and this may have prevented the iron acquiring a higher magnetisation. The fact of coils becoming heated when large currents are used is a bar to obtaining more than a few hundred C.-G.-S. units of magnetising force this way. We have previously shown (para. 130) that the magnetising field in C.-G.-S. units produced in the interior of a long narrow coil is equal to

$$\frac{\text{ampere-turns}}{\text{length of coil (centimetres)}} \times 1{\cdot}257$$

and therefore, in the case before us, in which the coil is 1ft. (30·48 centimetres) long, the magnetising force per 1,000 ampere-turns is

$$\frac{1257}{30{\cdot}48} = 41{\cdot}24 \text{ C.-G.-S. units};$$

and, therefore, for, say, 6,000 ampere-turns, which is about 200 ampere-turns per centimetre length of coil, and not far from the maximum exciting power possible to use with one layer of wire, we obtain a magnetising force of only $41{\cdot}24 \times 6$, or about 250 lines of force per square centimetre in the coil. If the number of layers of wire is increased, the current put through cannot be as strong as through a single layer, since the radiation of heat is slower, and a limit of exciting power is soon reached probably not much above 300 ampere-turns per centimetre length of coil. This prevents the investigation by the magnetising coil method of the direction taken by the magnetisation curve under very high magnetising forces—that is to say, whether it ever reaches a maximum height beyond which no additional force will increase it, and if this condition of things (which means *absolute* saturation of magnetism) does take place, whether the curve continues horizontal or shows an inclination to descend. If these points, which are of the greatest interest, are to be determined, recourse must be had to some other method of magnetising the iron, by which powerful fields can be brought to bear upon it.

Immediately on reaching the highest current considered safe for the insulation, the resistance was increased again, step by step, and readings taken on both instruments as before. This gave the descending curve of magnetisation which followed pretty closely the shape of the ascending curve, but at some distance higher from it. The residual magnetism in the core was calculated by the above formula from the magnetometer deflection after the current was switched off. This deflection being 10, the residual magnetisation was

$$\mathsf{I} = 1086 \times \cdot 1 = 108 \text{ C.-G.-S. units.}$$

This is only 9 per cent. of the total magnetisation acquired, and is due to the demagnetising effect of the core on itself, already spoken of.

In order to prove that the magnetometer has worked all right during the set of readings taken, as soon as the residual magnetism has been measured the core should be withdrawn from the coil and placed at a distance, upon which the needle of the magnometer should return accurately to zero. We shall next consider the magnetisation of a wire of annealed iron, of sufficient length to render the demagnetising effect inappreciable.

135. **Electro-Magnetisation of Annealed Iron Wire.**—In order to observe the magnetisation of a piece of iron when there is practically no demagnetising effect from its own poles, it has been stated that a rod or wire must be selected whose length is at least 300 times its diameter. Now, if the magnetic moment is measured by either of the two methods of Gauss already detailed (para. 125), the iron wire tested must be very small in diameter, as it is not convenient to test a very long wire by these methods. And the strength of pole developed in a wire of small diameter is necessarily very small even when the iron has acquired a high magnetisation, in consequence of which the magnetometer can only be deflected to very small angles. With a light mirror attached to the needle, the amplitude of the deflections can be considerably increased by a ray of light reflected from the mirror on to a screen, and

in this manner very small movements of the needle can be accurately observed. We are here, however, employing a magnetometer without a mirror attachment, and it is therefore advisable to arrange matters so that the needle is deflected to a considerable angle, say to 50 deg. or 60 deg., when the iron is near its highest magnetisation. The intermediate deflections can then be read with ease and accuracy. This may be effected conveniently by using a long core and placing it vertically, as shown at C (Fig. 116). As large a diameter as No. 11 B.W.G.

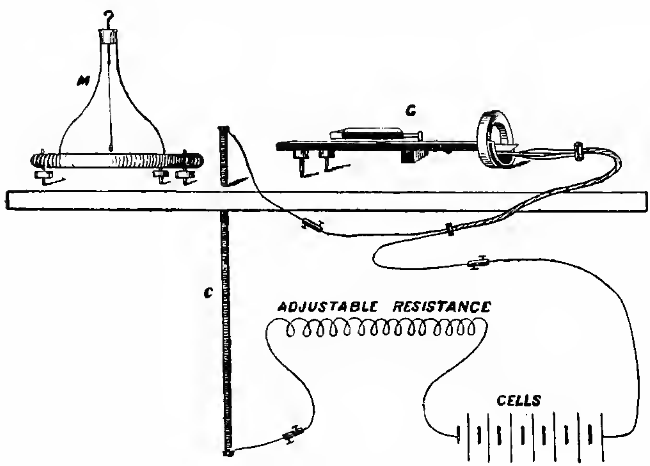


FIG. 116.

iron wire may be used for a core 3ft. long, and yet be over the necessary ratio of length to diameter. This gives sufficient sectional area of iron to develop, at high magnetisations, enough strength of pole to strongly deflect the needle, the top end of the core being within 20 centimetres from the needle. In this diagram, C is the magnetising coil wound round a brass tube, and fixed vertically by passing it through a hole in the table. The iron wire core of the same length as the tube should slip easily into it, and the upper end of the latter be fixed a little above the level

of the magnetometer needle, in order that the pole, which is formed at a little distance from the end, may act more nearly in a horizontal line with the needle. It has been pointed out by Sir William Thomson (in Papers on Electricity and Magnetism, p. 512) that as the iron becomes more magnetised the poles move up more towards the ends, but are always some slight distance off, in long thin cores, even when most strongly magnetised. It is the position of the poles and the variation of field along the length of a magnetised bar that

Fig. 117.

constitutes what is known as "magnetic distribution." With the magnetising coil vertical the changes in distribution or position of the poles during magnetisation produce minimum effect on the results. The upper pole now acts powerfully on the needle, and with a long core the action of the lower pole is very feeble. If we neglect the action of the lower pole, the intensity of field at the needle, placed r centimetres away from the upper pole, is

$$\frac{m}{r^2} \text{ C.-G.-S. units,}$$

where m is the strength of pole (*see* paras. 108 and 109). But where accuracy is required the action of the lower pole must be taken into account. Taking the point p (Fig. 117) as the centre of the magnetometer needle and SN as the magnetised core, it will be seen that the lower pole (R centimetres from p) would repel a unit north pole at p with a force of

$$\frac{m}{R^2} \text{ dynes.}$$

But this force is in an oblique direction, and the needle moving in a horizontal plane is only influenced by horizontal forces. It is therefore only the horizontal component of the above force that we have to deal with. Resolving by the parallelogram of forces we have by similar triangles

$$\text{horizontal component} : \frac{m}{R^2} :: r : R,$$

whence the horizontal force $= \dfrac{m\,r}{R^3}$ dynes.

The resultant force f at the needle is then given by the difference between the forces due to each pole, viz. :—

$$f = \frac{m}{r^2} - \frac{m\,r}{R^3} \text{ dynes.}$$

Now, the force in dynes on unit pole at a point is the same thing as the intensity of field at that point, and, therefore, the above value of f represents the strength of field at the needle centre due to both poles of the core. This field also acts on the needle at right angles to the earth's horizontal field H, and therefore (by para. 119)

$$f = H \tan \alpha,$$

where α is the angular deflection of needle. Glancing at the figure it will be noticed that the end of the magnetometer needle is viewed, the earth's meridian being supposed to be at right angles to the plane of the paper, and the magnetising coil C being placed east or west of the needle.

Equating the above values of f, and reducing by simple algebra, we have

$$m = \frac{\text{H} \tan \alpha \, r^2}{1 - \left(\frac{r}{\text{R}}\right)^3},$$

which gives the strength of each pole in C.-G.-S. units, and from which the magnetisation may be calculated by

$$\mathsf{I} = \frac{m}{a} \text{ C.-G.-S. units,}$$

where a is the sectional area of the core in square centimetres.

The cubed fraction in the denominator is the ratio of the distances of the poles of the core from the needle, and this fraction is small when the core is long and placed near to the needle. If the cube of this fraction is very small and is neglected it will be seen that the formula represents the action of the upper pole alone.

The manner of taking the test is precisely the same as that already detailed for the short iron rod. In the figure the adjustable resistance is shown conventionally, its practical form having already been detailed in para. 131 and Fig. 112.

The following practical example may serve as a guide to work by :—A brass tube of three millimetres bore and 90 centimetres long was first served round with an insulating covering of brown paper and then wrapped round carefully with No. 18 silk-covered copper wire. This took 657 turns of wire in all, or 7·3 turns per centimetre. Melted paraffin wax was brushed over the coil, and laid on quite hot so as to allow it time to soak well into the silk before solidifying. The current was obtained from a maximum number of 12 secondary cells through two adjustable resistances, one of German silver wire (Fig. 112) and the other of water, the latter being for the smaller alterations in resistance. Measurements of the current strength were taken on a Thomson-graded galvanometer G (Fig. 116), the manner of using which has been described. The controlling magnet was not used in any of the readings, and therefore the current was found by

$$\text{C} = \cdot 18 \, \frac{\text{D}}{\text{P}} \text{ amperes,}$$

where D is the deflection and P the position of the needle-box (para. 132). A curve was then plotted on squared paper from the readings in the same manner as described for the last curve, the vertical ordinates being magnetometer deflections and the horizontal ordinates the magnetising forces, calculated in ampere-turns *per foot.* The reason of this is that the last curve (Fig. 115) was plotted from readings taken with a magnetising coil one foot long, and therefore the magnetising forces in ampere-turns expressed on that curve are actually ampere-turns *per foot.* Now we wish to compare the rates of magnetisation of the two cores under test for a given series of magnetising forces, and therefore to be able to compare the two curves together we plot the magnetising forces exerted on the core at present under consideration in the same units as those employed on the previous core (para. 134), viz., ampere-turns per foot. We have already noted (para. 130) that the magnetising fore in C.-G.-S. units is equivalent to

$$\frac{\text{ampere-turns}}{\text{length of coil (centimetres)}} \times 1{\cdot}257\ ;$$

that is, the number of ampere-turns per centimetre multiplied by 1·257, from which it may be noticed that magnetising forces are proportional to the number of current-turns per unit of length. When comparing two or more coils of *the same length* the strength of field set up by them or their magnetising forces are proportional simply to their respective number of ampere-turns, but when dealing with coils of different lengths the simple statement of the number of ampere-turns on each offers no basis of comparison between them; we must know the number of ampere-turns for a certain definite length of coil in each. (We are referring to cylindrically-wound coils, as generally used for electro-magnets, not ring-shape coils, as in galvanometers.) For example, take two coils, each wound with one layer of wire of similar size and around bobbins of similar diameter; one bobbin being twice the length of the other. If the same current is passed through both, the number of ampere-turns in the former is twice that in the latter, and the length of wire wound on is also twice as much, but the magnetic field developed by

each, or their respective magnetising forces, are equal, for we have the same number of ampere-turns per unit of length in each. Again, take two coils, each wound with one layer of wire of similar size and around bobbins of similar length; one bobbin being twice the diameter of the other. If the same current is passed through both, the number of ampere-turns per unit of length is the same in each, and therefore the magnetic field developed by each is the same, although the length of wire wound on the former is twice that on the latter. And, further, if we take two exactly similar bobbins and wind any size wire on each, the magnetic fields developed are proportional to the number of ampere-turns on each; and if the current is the same in both, the fields are proportional simply to the number of turns or to the length of wire wound on each The remarks in para. 128 allude to this latter and more simple case.

We may put these facts very concisely in algebraical form by saying the magnetising force produced by a coil is proportional to

$$\text{Current} \times \frac{\text{total wire length}}{\text{coil diameter} \times \text{coil length}};$$

or the same law may be expressed in C.-G.-S. units by derivation from that already given in para. 130, viz.:—

$$\begin{array}{c}\text{Magnetising force}\\ \text{in C.-G.-S. units}\end{array} = \frac{\text{ampere-turns}}{\text{coil length (centimetres)}} \times 1{\cdot}257,$$

where the constant 1·257 is the ratio $\frac{4\,\pi}{10}$, as explained in that paragraph. For, the number of turns in any coil is equal to the total length of wire, divided by the mean length per turn, and the mean length per turn is equal to the mean diameter of the coil multiplied by π (3·1416).

Therefore, cancelling out the constant π, we may write the above in the form

$$\begin{array}{c}\text{Magnetising force}\\ \text{in C.-G.-S. units}\end{array} = \text{Amperes} \times \frac{\text{total wire length}}{\text{coil length} \times \text{coil diameter}} \times \frac{4}{10},$$

all measurements of length being in centimetres. From this

it is clear that it is only when the denominator of the expression—that is, the *mean surface* of different size cylindrical coils—is the same that we can compare the strengths of field produced by each by the product of the current and the length of wire, in other words, by the number of ampere-feet on each.

To return, however, to the magnetisation test. The number of turns per foot was 222·5, and the magnetising force was, therefore,

$$\text{Amperes} \times 222{\cdot}5,$$
$$= {\cdot}18\,\frac{\text{D}}{\text{P}} \times 222{\cdot}5 \text{ ampere-turns per foot.}$$

Reducing all galvanometer deflections (D) to position **32** (P = 32), as in the last example, and multiplying up, we have

$$\text{Ampere-turns per foot} = 1\tfrac{1}{4}\ \text{D}\ ;$$

all that was necessary, therefore, was to add on to each deflection (reduced to 32) one-quarter of its value. These results give the magnetising forces in ampere-turns per foot, due to the current in the coil; but we have an additional magnetising force constantly acting on the core, which must be taken account of. The core, being vertical, is acted upon by the vertical component of the earth's field. Taking the Kew Observatory data given in para. 110, and calculating out the value of the above, we have

$$\text{Earth's vertical field} = \text{H} \times \text{tangent of angle of dip.}$$

The angle of dip or inclination is 67° 37′, and its tangent = 2·4282. Therefore

$$\text{Vertical field} = {\cdot}1810 \times 2{\cdot}4282 = {\cdot}4394 \text{ C.-G.-S. units.}$$

This must be expressed in ampere-turns per foot, in order to add it to the magnetising forces due to the coil. We can do this from the expression

$$\text{Field in C.-G.-S. units} = 1{\cdot}257 \times \text{ampere-turns per centimetre.}$$

Now, 1 foot = 30·48 centimetres, and therefore the number of ampere-turns per centimetre is equal to

$$\frac{\text{Ampere-turns per foot}}{30{\cdot}48}.$$

Substituting this value, and dividing out, we have

$$\text{Ampere-turns per foot} = \text{field in C.-G.-S. units} \times 24\tfrac{1}{4}.$$

The vertical force of the earth is, therefore, in this latitude, equal to a magnetising force of

$$\cdot 4394 \times 24\tfrac{1}{4} = 10\cdot 66 \text{ ampere-turns per foot,}$$

and is added to the magnetising forces set up by the coil to give the total magnetising force. We say *added*, because the

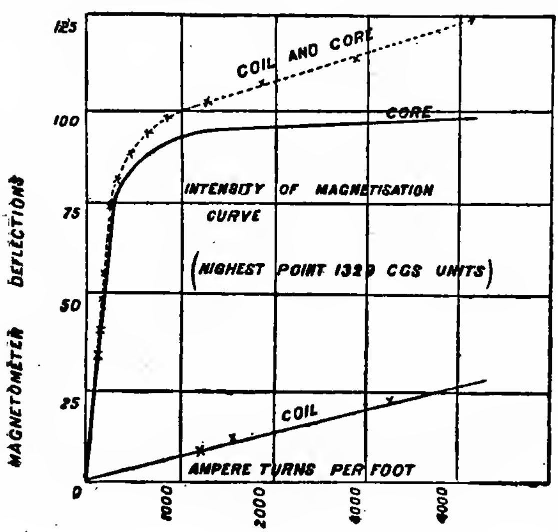


FIG. 118.

current was purposely passed round the coil in the direction to cause the lower end of the core to become a north pole, and the earth's vertical force has the same effect. Had the current been in the other direction, the earth's force would have been subtracted. The usual method of dealing with this force is to neutralise it by winding another coil outside the first coil. This extra coil is usually of fine wire, and a current is passed through it just sufficient to set up a field exactly equal and opposite to

that of the earth, and this current is kept on continuously during the magnetisation test, neutralising the vertical field.

The iron core, which was mechanically very soft and carefully annealed, was 90 centimetres long and 2·6 millimetres diameter. The length was, therefore, 346 times the diameter, and over Prof. Ewing's minimum limit for negligible demagnetising effect. In consequence of this it will be noticed that the rise in magnetisation is much more rapid at the commencement of the curve (Fig. 118) than with the former short iron core. The vertical ordinates of the two curves do not allow of comparison, since the cores are of different dimensions. This comparison must be made by the intensity of magnetisation, which can be calculated by the formula stated above. We must first find the value of m.

The distance (r) of the upper pole from the needle was 20 centimetres, and the length (l) of core being known (90 centimetres), the distance (R) of the lower pole can be found by

$$R = \sqrt{r^2 + l^2},$$

R being the hypothenuse of a right-angle triangle, of which r and l are the two other sides ("Euclid," I., 47). Working out these values we get for the denominator of the above expression ·9898, and for the numerator 72 tan a. We have, therefore,

$$m = 72{\cdot}73 \tan a.$$

Now the sectional area of core is equal to

$$\frac{\pi d^2}{4} = {\cdot}7854 \times ({\cdot}26)^2 = {\cdot}053 \text{ sq. cms.},$$

and therefore the intensity of magnetisation is equal to

$$\frac{m}{a} = \frac{72{\cdot}73 \tan a}{{\cdot}053} = 1{,}370 \tan a.$$

We can now find the magnetisation at any given point on the curve. Take the highest point, viz., 97 divisions:—

$$I = 1{,}370 \times {\cdot}97 = 1{,}329 \text{ C.-G.-S. units.}$$

On switching off the current the magnetometer remained steady at 45 divisions; hence the residual magnetisation was

$$1{,}370 \times {\cdot}45 = 616 \text{ C.-G.-S. units,}$$

or 46 per cent. of the maximum magnetisation acquired. When the current is suddenly interrupted in this manner the residual magnetism is not so great as when the current is gradually reduced to zero. In soft annealed iron wires, whose relation between length and diameter was similar to the above, Prof. Ewing found the residual magnetism some 85 per cent. of the maximum ; and as much as 90 and 93 per cent. in other specimens of soft iron ("Researches in Magnetism," *Phil. Trans.*, 1885).

136. Comparison of Magnetisation Curves.—The two curves obtained have been plotted with magnetometer deflections as ordinates, but they cannot be compared by these deflections, as they were taken by different methods and with cores of different sectional area. We shall now compare them by plotting them with the intensities of magnetisation as ordinates, and at the same time express the magnetising force in C.-G.-S. units instead of ampere-turns per foot. For every 1,000 ampere-turns per foot we have 41·24 C.-G.-S. units (para. 134), and by inspection of each curve we find the magnetometer reading ($\tan \alpha$) corresponding to each 1,000 ampere-turns per foot and intermediate points. The intensity of magnetisation is derived from the magnetometer reading by

$$\mathsf{I} = 1086 \tan \alpha$$

in the first curve (para. 134), and by

$$\mathsf{I} = 1370 \tan \alpha$$

in the latter.

For example, in the latter curve we find by inspection that for 1,000 ampere-turns per foot (equal to 41·24 C.-G.-S. units) the deflection is 92, and therefore the magnetisation is $1370 \times {\cdot}92 = 1260$ C.-G.-S. units. Similarly, for 400 ampere-turns per foot (equal to 16·5 C.-G.-S. units) the deflection is 80 and the magnetisation $1370 \times {\cdot}8 = 1096$ C.-G.-S. units. Taking several points this way, and plotting them as in Fig. 119, we are able to observe the relative magnetisation of the two cores. The main points of comparison are the relative magnetisations acquired under any given magnetising force and the magnetic "reten-

tiveness" of each core. It should be borne in mind that the top curve is that taken with a core whose dimensions satisfy the condition of "endlessness," and is a soft annealed wire, while the lower curve is that from a short iron rod of soft iron, but unannealed. The marked difference in the rise of the two curves is partly due to the slightly different nature of the iron, but *chiefly* due to the dimensions of the cores. This is very clearly shown by Prof. Ewing (to whose results the above are similar) in his experiments. He cut from the *same piece* of iron wire (of 1·58 millimetre diameter) different lengths, whose ratios of length to diameter varied from 50 to 300. The demagnetising effect and slow magnetisation were, therefore, proved to be due to the ends in short lengths.

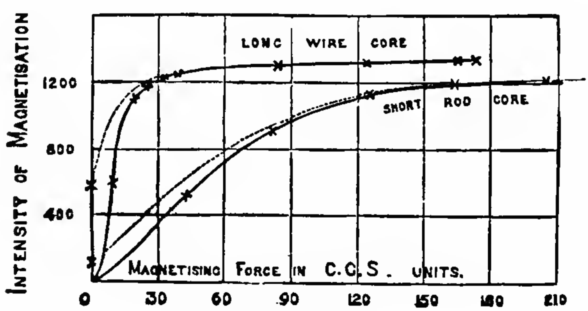


Fig. 119.

In the above there was practically no demagnetising effect set up by the poles of the long wire core, and hence the full magnetising force of the coil could take effect upon it. With the short rod core, however, the poles caused a considerable reverse field of force, through the metal (para. 133) neutralising part of the field of the magnetising coil, and preventing the iron from becoming magnetised. This accounts for the very little residual magnetism left, being only about 9 per cent. of the maximum, for, when the current in the coil was reduced to zero the poles of the core exerted their reverse field, tending to neutralise the magnetism acquired. On the other hand, with

the long wire core the poles could exert no demagnetising effect, and hence the iron, though softer, retained a much greater percentage of magnetism. These facts indicate clearly that magnetisation and residual magnetism depend as much on the shape as on the kind of iron. While it is always best to use soft iron to obtain the highest magnetisation for a given magnetising force, it is advisable in the construction of electro-magnets where the residual magnetism is to be minimised to make the core short and thick rather than long and thin. The cores of electro-magnets of fast-speed telegraph instruments have of late years been constructed more of the "squat" shape, this having been found to allow of more rapid signalling than the original tall, thin cores.

137. Magnetic Susceptibility.—As Science moves onward she draws in her train an ever-increasing number of newly-coined or adopted words and names, to each of which she assigns a definite *rôle*—that of representing with rigid exactness some new idea or phenomenon, or perhaps of being the more concise representative of some already known but hitherto undefined property of matter. Perhaps in no paths of discovery has she so far perpetuated in this manner the names of those who have worked laboriously in her cause as in electrical and allied researches, in which such famous names as Volta, Ampère, Ohm, Coulomb, Faraday, Joule, and Watt are constantly in use by those who are engaged in developing the many practical applications turning on their discoveries. It sometimes happens that Science presses into her service words which have been more or less in the familiar use of common parlance. When, however, such words are "told off" to serve her, she requires that, whatever variety of garb they may choose to wear in their capacity as civilians, one distinctive uniform only must be worn when they are enlisted in her corps and on duty in her service. In other words, Science requires that definite ideas shall be represented by special terms set apart exclusively for their use. As instances, we might cite such words as force, strength, mass, intensity, power, work, energy, capacity, &c., which all have considerable flexibility of meaning in ordinary

use, but only one distinctive meaning in their scientific application.

The word "susceptibility," which we are about to consider, has been told off to express a state or condition of iron in which it is more or less "susceptible" of becoming magnetised under the action of electro-magnetic force. In the familiar use of the word we speak of "susceptibility to pain," "susceptible of impressions," "susceptible to charms," &c.; and, turning to see what Johnson says, we find he quotes a sentence from "Wotton" to illustrate its meaning as follows:—"He moulded him platonically to his own idea, delighting first in the choice of materials, because he found him *susceptible* of good form." A noted judge, prominent in electrical litigation, is reported to have said on one occasion, "No doubt the words used are *susceptible* of two constructions." Put into the simplest language, we may say that great or little susceptibility conveys the idea of the ease or difficulty with which an acting cause produces a corresponding effect. When the subject operated upon possesses great susceptibility a given acting cause can produce a great effect, but when it has little susceptibility the same acting cause can produce but little effect. The subject operated upon in our present case is iron, the acting cause magnetising force, and the effect the magnetisation produced. Iron is said to have a greater maximum *magnetic susceptibility* than steel, because under a given magnetising force it acquires a higher intensity of magnetisation. Also, the magnetic susceptibility of iron or steel varies according to the state of magnetisation. On applying a gradually-increasing magnetising force to iron its susceptibility is at first small, then increases very rapidly to a maximum, and then falls again almost to zero. This may be noticed in the curves already worked out.

We may now define magnetic susceptibility (usually denoted by the Greek letter κ) as the ratio between I, the intensity of magnetisation acquired, and H, the acting magnetising force, and write it

$$\kappa = \frac{\mathsf{I}}{\mathsf{H}}.$$

For example, taking the higher curve in the last figure it will be noticed that for a magnetising force of 10 C.-G.-S. units the magnetisation acquired is 800, and therefore the susceptibility is 80. This is at the steepest part of the curve, and is therefore the maximum susceptibility in this specimen. As the iron becomes practically "saturated" at the sharp bend of the curve the susceptibility is reduced to $\frac{1200}{30}=40$, and at the highest magnetising force employed (170 C.-G.-S. units) it becomes very small, viz., $\frac{1330}{170}=8$ nearly.

Prof. Ewing found the susceptibility of a specimen of very soft iron to reach up to 280, and when mechanical vibrations were applied to the same specimen while under the magnetising force its susceptibility increased to as high a value as 1,600.

One important point must be noted. Any piece of iron, the susceptibility of which is to be determined, must comply in dimensions with the condition of "endlessness" spoken of in para. 133, otherwise the exact magnetising force brought to bear upon it cannot be computed. For instance, from the lower curve in the last figure the susceptibility of that specimen of iron cannot be derived, because the dimensions of the core are such that a demagnetising force of unknown value is acting on it in addition to the force due to the coil.

138. Further Developments of the Subject.—Before leaving this subject we shall indicate in what direction the student may experiment with advantage. Besides the acquaintance he will gain with the subject by trying cores of various specimens of iron and steel of different dimensions, he should carry the magnetisation of some specimens through a complete phase or cycle; that is, starting by increasing the current from zero, the ordinary curve of magnetisation rises from 0 (Fig. 120). After carrying this on until the core is fairly saturated, the current is gradually reduced to zero. The curve, as we have seen, will not return to the point of origin by reason of the residual magnetism O A. The current in the magnetising coil is then reversed, and precisely the same operation repeated—viz.,

gradual increase of current to fair saturation, and then decrease to zero. This brings us to the point B, the residual magnetism being O B. Now the current is reversed again, and gradually increased until the highest point in the curve is again reached. The core has then been led through a complete cycle of magnetisation. The valuable suggestion made by Mr. Sidney Evershed (*The Electrician*, Nov. 9, 1888) for eliminating the effect of the magnetising coil on the magnetometer, simultaneously with the

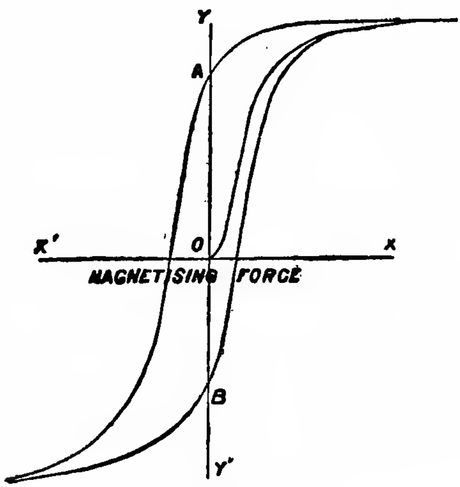


FIG. 120.

test, by winding a few turns of the said coil backwards round the magnetometer needle at such a distance as to just neutralise its own effect, should also be taken advantage of. This would allow of seeing at once by the magnetometer deflections when the core approached saturation, and also save the time of plotting two curves.

The core having been led through a cycle of magnetisation in the manner indicated, we have a completely closed curve, the area enclosed by which is proportional to the amount of work spent per unit volume of iron in conducting it through the cycle.

It is evident that the cause of the cyclic curve being of this general shape and enclosing an area is due to the fact that the descending curve does not coincide with the ascending curve, and this latter fact is due to residual magnetism of the iron. In other words, the magnetisation of the iron follows at a distance or "lags behind" the magnetising force acting on it. Prof. Ewing, in a set of exhaustive experiments on this property of iron, carried out some five or six years ago at the Physical Laboratory of the University of Tokio, Japan, calls the phenomenon "*Hysteresis*," from ὑστερέω, to lag behind, and defines it as "the lagging behind of changes in magnetic intensity to changes in magnetising force" ("Researches in Magnetism," *Phil. Trans.*, 1885). In the same Paper we find the pregnant remark, "The existence of residual magnetism, when the field is reduced to zero, is, in fact, only one case of an action which occurs whenever the field is varied in any way, viz., a tendency to persistence of previous magnetic state."

In all electro-magnetic apparatus where the magnetising force is alternating in direction, or where an iron mass moves periodically through alternate fields, the iron undergoes this cyclic change, and we have the phenomenon of hysteresis. In dynamo and motor armatures, and in the cores of "transformers" or "converters" used in distributing electrical energy by the alternate current system, the loss of energy by hysteresis is an important point affecting more or less their working efficiency. The dissipated energy due to this cause takes the form of heat, resulting in a rise of temperature in the iron. In the earlier forms of "transformers" the iron cores became perfectly hot from this cause, showing the necessity for working with a weaker field through the iron.

Mr. Gisbert Kapp, in his Paper on "Alternate-Current Transformers," before the Society of Telegraph-Engineers and Electricians (*The Electrician*, February 10th, 1888), in describing the experiments conducted by himself and Mr. W. H. Snell in the design of their well-known transformer, observes that, owing to the rise of temperature of the iron due to hysteresis, the "induction" through the iron had to be reduced to somewhat less than 10,000 C.-G.-S. units (*see* paras.

139 and 140), which may be considered as about one-third the maximum induction at which it is practically possible to work dynamo armatures constructed of the softest wrought iron. It is not within the scope of the present work to treat of the above-mentioned machines; the student who is pursuing the subject should, however, consult the works already referred to, in addition to which he will find the subject very clearly treated in a "Note on Magnetic Hysteresis," by Dr. J. A. Fleming (*The Electrician*, September 14, 1888). In furtherance of the subject of magnetic susceptibility, a Paper by Mr. R. Shida (*Proc.* Royal Soc., 1883), and one by Mr. A. Tanakadaté on the "Mean Intensity of Magnetism of Soft Iron Bars of Various Lengths in a Uniform Magnetic Field," read before Section A of the Bath meeting of the British Association (*The Electrician*, November 2, 1888), will be found valuable sources of information.

139. Electro-Magnetic Induction.—We shall conclude this section with a consideration of electro-magnetic induction and magnetic permeability, both of which are closely allied to the foregoing subject of magnetisation, and in such constant use in descriptions of, and discussions on, electro-magnetic appliances as to make it important for the student to have a clear conception of their meanings. At the same time, consistently with making the subject intelligible, we shall dwell on it as briefly as possible.

At the outset, the relation which "magnetic induction" bears to "intensity of magnetisation" should be clearly understood. We may state this concisely by saying that intensity of magnetisation is proportional simply to the number of lines of force per unit area passing through the iron *due to its own acquired magnetism*, while magnetic induction is proportional to the number of lines per unit area passing through the iron, due to its own acquired magnetism, *plus those due to the magnetising field.* The latter is the more practically useful means of measurement, since, in practice, we necessarily concern ourselves with the *combined magnetic effect* of the magnetised iron and the electro-magnetic field causing its magnetisation. Recalling

what was said in para. 133, it will be remembered that through the mass of any piece of iron which has been magnetised to a strength of pole equal to m C.-G.-S. units, and whose dimensions satisfy the condition of "endlessness," there are $4\pi m$ lines running from pole to pole (south to north). The condition of "endlessness" ensures the absence of any demagnetising lines (running from north to south, or in a contrary direction through the metal) which would make the actual number of lines through such a piece of magnetised iron less than $4\pi m$.

Such being the case, we have in a piece of iron of a square centimetres (uniform) cross-section $\frac{4\pi m}{a}$ lines per square centimetre passing through its mass and *due to its own acquired magnetism.* Now, the intensity of magnetisation (I) is found by

$$\mathsf{I} = \frac{m}{a},$$

putting which value in the above we find that

$$4\pi\mathsf{I} = \text{lines per sq. cm. due to acquired magnetism};$$

and, therefore,

$$\mathsf{I} = \text{lines per sq. cm. due to acquired magnetism} \times \frac{1}{4\pi}.$$

This proves the statement made above, that the intensity of magnetisation is proportional to the lines of force per square centimetre through the iron, due *alone* to the magnetism acquired. The same fact will have been noticed in the practical examples already detailed, in which the magnetising force of the coil was *subtracted* from the combined magnetic effect of the coil and core together before calculating the intensity of magnetisation. Similarly, we have seen that the strength of a *permanent* magnet may be expressed in units of intensity of magnetisation, since the term only involves the lines of force due to its own magnetism, or the strength of pole of the magnet *itself;* but the same cannot be expressed in units of magnetic induction, since this term implies the

presence of a magnetising force, and must include the value of the same.

Now, to understand the precise meaning of magnetic induction, let us take a simple cylindrical coil of wire as at H (Fig. 121). On passing a known current through it, a magnetic field of a definite intensity is set up in its interior. We have

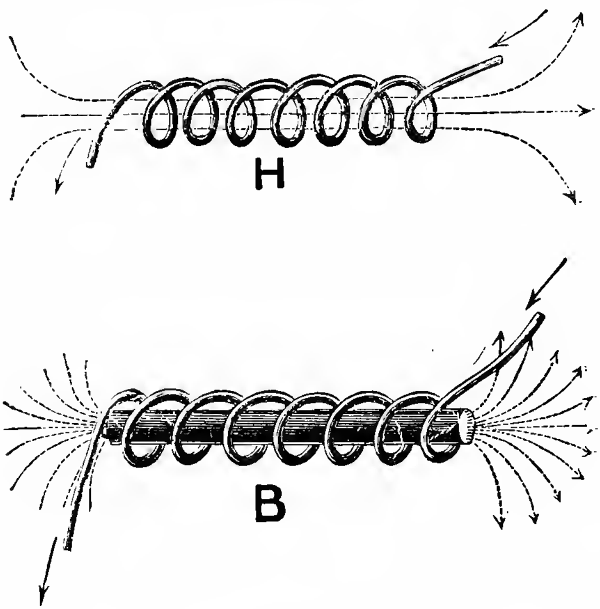


FIG. 121.

then a magnetising coil—that is, one capable of magnetising any magnetic metal placed within it. We have shown that the field intensity in the coil, or, what is the same thing, the lines per square centimetre, can be easily calculated. Suppose there are **H** lines per square centimetre through the coil. Now, on putting an iron core inside, as at B, we find that the intensity of field becomes very much stronger than before; in

fact, the presence of iron in the coil has enormously increased the number of lines. Now, what is termed "magnetic induction," or sometimes simply "induction," and denoted by the letter B, is the strength of field which exists when the iron is put in—that is, the number of lines per square centimetre. It must be remembered, however, that the part of this field B which we called H existed *before* the iron was put in, and therefore $\mathsf{B} - \mathsf{H}$ lines have been *created* by adding the iron. We say created because they did not exist in the iron before it was put in the coil, neither did they exist in the coil space before. The lines $\mathsf{B} - \mathsf{H}$ are therefore those due *alone* to the magnetism acquired by the iron, and these we have found above to be equal to $4 \pi \mathsf{I}$. We therefore arrive at an equation expressing the fundamental relation between "magnetisation" and "induction," viz.,

$$\mathsf{B} - \mathsf{H} = 4 \pi \mathsf{I},$$

from which, if we know the magnetising force H and the magnetisation I, we can find the induction by

$$\mathsf{B} = 4 \pi \mathsf{I} + \mathsf{H}.$$

For example, by inspection of the upper curve in Fig. 119 we find that the magnetisation was 1,280 units for a magnetising force of 54 units. Hence the magnetic induction for this force was

$$4 \pi\, 1{,}280 + 54 = 16{,}144 \text{ lines per square centimetre,}$$

of which 54 are due to the magnetising coil and 16,090 to the magnetism acquired by the iron. Working out a few values in the above manner we obtain data to plot a fresh curve (Fig. 122) for the same core, the vertical ordinates being now in C.-G.-S. units of magnetic induction or lines of force per square centimetre through the iron. This shows the connection between induction and magnetisation, and how one may be deduced from the other. It is possible, however, that the student may find some difficulty in conceiving the physical meaning of the formula as applied to the reverse operation, viz., that of deduc-

ing the magnetisation from the magnetic induction. From what we have shown above, the formula is evidently :—

$$\mathsf{I} = \frac{\mathsf{B} - \mathsf{H}}{4\pi}.$$

Now, suppose the value of B in a certain *ring* of iron under a given magnetising force H has been ascertained. It is simple enough to put these values into the above formula and so calculate I; but the physical meaning is not so simple to follow, for by I is meant the pole strength per unit area of metal, or

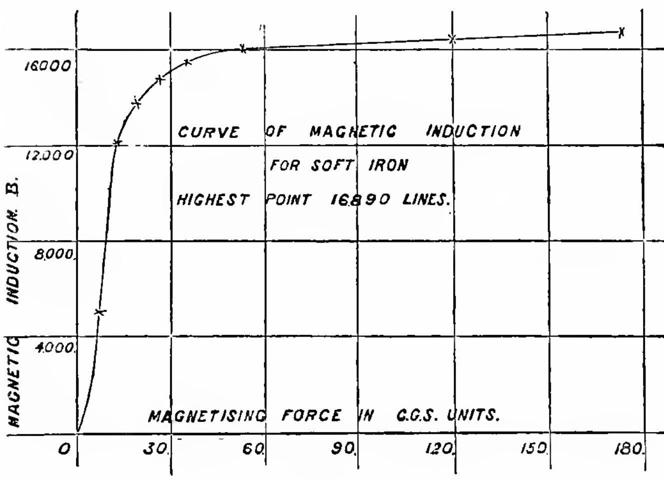


Fig. 122.

moment per unit volume, while the piece of iron under test is ring-shaped, and, therefore, has no poles and no magnetic moment. The explanation is that for a given pole strength per unit area, the mass of the iron is in a certain definite condition magnetically, which can be represented as equivalent to a definite field or number of lines per square centimetre, running through its substance. Now, it is clear that this identical magnetic condition of the iron, as regards the number of lines through it, can exist equally well when the iron forms a closed

circuit, and exhibits no polarity at all; indeed, we have already shown that the internal field equivalent to a magnetisation **I** is represented by 4π **I** lines, and therefore the formula is quite intelligible. With a ring-shaped iron core the magnetisation cannot be determined by a magnetometer, as there are no poles to act on the needle, but it can be calculated in the above way from a direct experimental determination of the magnetic induction.

140. Practical and Absolute Magnetic Saturation.—It will be noticed in the above curve that magnetic induction in the iron, up to 12,000 lines, takes place very rapidly under the action of 10 or 12 C.-G.-S. units of magnetising force, and that as the force is increased beyond this point induction occurs more slowly. For instance, double the force (24 units) only produces 14,700 lines, or an increase of a little over one-fifth the field. Again, three times the force (36 units) only produces 15,700 lines, or an increase of about one-third the field. And it will be seen by the curve that any further increase in the force produces but slight increase in the induction; in fact, for practical purposes, it would be very uneconomical to expend 180 units of magnetising force to produce 16,900 lines, when 16,000 could be produced with less than one-quarter the force. It is for this reason that the top bend of the curve just before it becomes almost horizontal is taken as the practical limit of saturation, and in soft iron may be taken as about 16,000 lines.

In the best quality of iron now obtainable, such as Swedish charcoal iron or well-annealed wrought iron, the induction curve rises quicker, and reaches considerably higher values. For dynamo armatures built with iron of this quality a fair average saturation met with in good machines ranges from about 18,000 to 20,000 C.-G.-S. lines, or sometimes higher. The point of "absolute saturation" is of purely theoretical value, and, of course, means the point at which the curve ceases altogether to rise. It would appear that there could be no absolute saturation point for magnetic induction, because the magnetising force is one of its constituents, and, therefore, it would

seem probable that induction would always increase with an increase in magnetising force. To determine this question evidently requires the employment of exceedingly high magnetising forces, to obtain which the magnetising coil, as applied directly to the specimen of iron under test, is out of the question. The researches of Prof. J. A. Ewing and Mr. William Low in this direction furnish by far the most advanced experimental data on the subject, inasmuch as these able observers have for some time brought to bear upon the specimens of iron under test far higher magnetising forces than have been hitherto employed. Their method consists in placing the piece of iron between the poles of a powerful electro-magnet, so as to form a complete magnetic circuit, and thus force a large number of lines through it. To increase the density of lines through the test piece it is thinned at the centre by turning down to a very small diameter, while its two sides extend outwards, cone-like, to fit the pole-pieces. This gives the test piece the shape of a reel or bobbin, of which the thin central portion is called the neck. By this means, as communicated to the Royal Society in their recent Paper (November 22, 1888), the authors have forced the magnetic induction in wrought iron to the enormous value of 45,350 C.-G.-S. units, and in cast iron to 31,760, and find that there is no appearance of a limit to the extent to which magnetic induction may be raised. To show what an advance this is, a remark of Drs. J. and E. Hopkinson in their able Paper on Dynamo-Electric Machines in 1886 *(Phil Trans.)*, may be quoted, "No published experiments exist giving the magnetising force required to produce the induction here observed in the armature core amounting to a maximum of 20,000 per square centimetre."

The direct measurement of the induction was made in the usual way; the central neck, where the induction is greatest, was wound with a coil of wire connected to a ballistic galvanometer, and the "swing" on the latter observed when the iron bobbin was turned round through half a revolution, so as to face the opposite poles of the electro-magnet. As this completely reverses the direction of magnetisation, the deflection

is proportional to twice the induction in the bobbin. We shall refer again to the ballistic galvanometer and the method of calibrating it in absolute units by the use of earth-coils. The measurement of the exact value of the magnetising force (necessary in order to calculate the intensity of magnetisation) was attended with great difficulty, and it was not until the Paper referred to above that the authors were able to speak with confidence of its determination. The method was to wind a second coil at a certain measured distance concentrically round the first, and to subtract the deflection obtained with the first coil from that obtained with the second, the difference being proportional to the field within the air space between the two coils. To investigate how nearly this field approximated to the magnetising force acting on the metal, was a determination of great difficulty, but the authors say in the above Paper that by modifying the form of the cone-shaped sides of the iron bobbin a uniform field was obtained in the central neck, and the magnetising force within the neck was sensibly the same as that in the air immediately around it.

Having established the correctness of the determinations of the magnetising forces employed it was found that for wrought iron the intensity of magnetisation remained stationary at 1,700 C.-G.-S. units, while the magnetising force was varied between the enormous values of 2,000 to 20,000 C.-G.-S. units, showing that apparently a point of "absolute" saturation as regards magnetisation may be reached. This saturation point for cast iron is found by the authors to be 1,240 units. The point of "practical" saturation for soft iron will be seen by the curve obtained above experimentally (Fig. 119) to be about 1,300 C.-G.-S. units.

141. Kapp Lines.—At this stage it may be well to allude to the English unit of magnetic induction introduced by Mr. Gisbert Kapp, and now largely in use by manufacturers of electrical machinery. A new unit line of force, equal to 6,000 C.-G.-S. lines, is adopted, and the sectional area of iron is taken in square inches instead of square centimetres. The new unit of induction is therefore one of these assumed lines per square

inch section, and is generally known as one Kapp line per square inch, or one English unit of induction.

The number 6,000 is composed of two factors, viz., 60, which enables the formula for the E.M.F. produced in a rotating armature to be expressed in revolutions per minute instead of per second; and the factor 100, which brings the measurement of induction down to figures in the tens instead of thousands. Now, if we divide 6,000 by 6·4514 (the number of square centimetres in one square inch), we have the number of C.-G.-S.

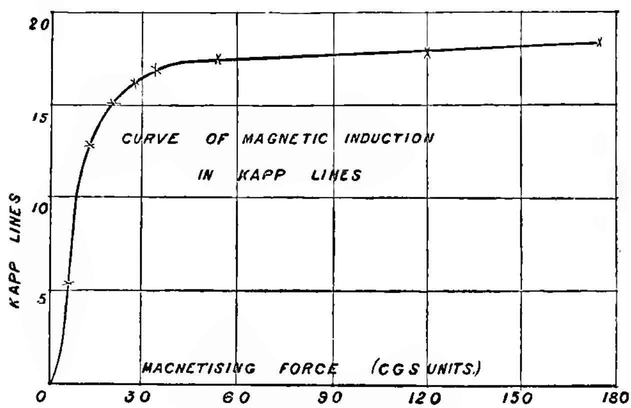


Fig. 123.

lines per square centimetre equivalent to one Kapp line per square inch = 930·03, or say 930; therefore,

1 Kapp line per sq. in. = 930 C.-G.-S. lines per sq. cm.,

or 1 English unit = 930 C.-G.-S. units of magnetic induction.

For the purpose of comparison the vertical ordinates of the last curve have been reduced to Kapp units by dividing the values of B by 930 (Fig. 123). The highest induction arrived at in this specimen of soft iron is seen to be 18 lines, and the practical saturation 17·2 lines (16,000 C.-G.-S. lines). In the previous paragraph we noted that there was no absolute limit

to induction, and that 20,000 C.-G.-S. units (= 21·5 Kapp units) might be considered a fair average induction in machines whose armatures were of well-annealed wrought iron. The extremely high value reached by Prof. Ewing of 45,350 C.-G.-S. lines with wrought iron is equivalent to 48·7 Kapp lines. One or two examples quoted from descriptions of dynamos in which the induction is expressed in these units may render their practical application intelligible. For instance, in *The Electrician*, Vol. XX., page 35, we find :—

"The cross-sectional area of the magnet bars is 30 square inches, and the total cross-sectional area of actual iron in the armature is 20 square inches. The total excitation of field-magnets at full load is 12,130 ampere-turns, and the total strength of useful field is 378 lines (English measure). The density in the magnet cores is therefore 12·6 lines per square inch, and in the armature core 18·9 lines per square inch."

Again, on page 162, in the description of another machine : "The field strength in the core, according to the figures supplied us, works out at a very high figure, viz., to 23·6 lines per square inch." These calculations are on the basis of Mr. Kapp's formulæ given in his Paper on "The Predetermination of the Characteristics of Dynamos" (*Journal* Soc. Tel. Eng., No. 64), and take into account magnetic resistance and leakage. In that Paper Mr. Kapp gives the average practical saturation values of induction derived from numerous data and experiments on machines, pointing out that the figures given are by no means the highest obtainable with exceptionably good iron. These are as follows :—Armatures built of well-annealed charcoal iron wire 25 lines per square inch, armatures built of discs of the same iron 22 lines, field-magnets of hammered scrap iron 18 lines. Multiplying by 930 we find these are equivalent to 23,250, 20,460, and 16,740 C.-G.-S. lines of induction respectively. Considerable exception to these English units was taken by scientists at the time of their introduction two years ago, chiefly on the ground of the necessary translation of coefficients used with formulæ based on the inch-minute system into their value on the centimetre-

second system. Mr. Kapp's reply to this and other objections (Society of Telegraph-Engineers' meeting, Dec. 2nd, 1886) was as follows:—"I use minutes because everybody is accustomed to counting revolutions per minute, and anybody quite unacquainted with French measure can work my formulæ, and would at the same time see what he is doing. The figures are of reasonable magnitude, and present to those who use them a definite meaning. They know what is meant by 17 lines to the square inch, but if we talk of 15,800 to the square centimetre a greater mental effort is required to grasp the meaning. . . . The translation from it to C.-G.-S. units is really not such a difficult matter as we are told, and with a little training one could get to manage either system equally well; but we have to talk to our workmen, and then we could not use grammes, centimetres, and seconds. We must give them figures in the usual English measure, and therefore it saves labour if we make the calculations in a system where the figures are of reasonable magnitude and directly applicable to the various purposes of the workshop."

142. Magnetic Permeability—To denote the degree to which iron and other magnetic metals permit lines of magnetic induction to *permeate* through their substance, under the action of a given magnetising force, the term "*permeability*" has been applied. The permeability of a specimen of iron is said to be high if it will permit the flow of a large magnetic induction through its mass for the expenditure of a small magnetising force, and *vice versâ*. In other words, magnetic permeability is the ratio between magnetic induction B and magnetising force H, and is usually denoted by the Greek letter μ. By inspection of the curves it will be seen that this property, like that of susceptibility, is not a constant quantity for any given kind of iron, but depends on its magnetic condition. For instance, in the curve given (Fig. 122) when the induction is between the limits of 4,000 and 12,000 C.-G.-S. lines, the permeability is fairly constant at about 800, but gradually falls to 300 at 16,000 lines, and ultimately to 100. In softer specimens of iron the permeability may reach 3,000 or 4,000.

The meaning of these figures may be better appreciated by considering them as indicating the permeability of iron with reference to that of air. In a magnetising coil, with no iron present, the induction produced in the air space in the interior is identically the same as the magnetising field, and, therefore, the ratio of the two, *i.e.*, the magnetic permeability of air, is equal to unity. The number indicating the permeability of iron is therefore the number of times its permeability exceeds that of air. When iron is acted upon by an alternating or increasing and diminishing magnetising force, its permeability, when not near saturation, is constant. The parallelism of the ascending and descending lines in the cyclic curve of magnetisation (Fig. 120) shows the susceptibility to be constant for low magnetising forces, and it follows that the permeability is constant. The exact relation between permeability and susceptibility may be found by taking the fundamental formula given in para. 139—

$$\mathsf{B} = 4\,\pi\,\mathsf{I} + \mathsf{H},$$

and dividing each side by H; we then have

$$\mu = 4\,\pi\,\kappa + 1,$$

or $$\text{Permeability} = 1 + (\text{Susceptibility} \times 12{\cdot}57).$$

We may regard the term permeability as meaning the conductivity of iron to lines of force, in the same way as we speak of the conductivity of iron to the passage of electric currents; with this important difference, however, that while the electrical conductivity of iron at a given temperature remains constant for all intensities of current, its magnetic permeability decreases to a very marked extent as the intensity of magnetic-field or number of lines through it is increased.

If we pass an electric current through a piece of iron wire, and gradually increase the intensity of current until the wire is red-hot, the conductivity of the metal will undergo a considerable change. As soon as the intensity of current is sufficient to cause the slightest rise in temperature of the iron, the conductivity of the latter commences to diminish, and

beyond this point any increase in the flow of current causes a rapid diminution of the conductivity. This decrease of conductivity in an electrical circuit, on increasing the intensity of current through the conductor, is somewhat analogous to the decrease of permeability in a magnetic circuit, on increasing the intensity of magnetic flow, or induction through the iron.

Section III.—Magnetic Fields of Coils and Solenoids.

143. Preliminary.—The phenomenon which may be considered the most important and fundamental to be appreciated

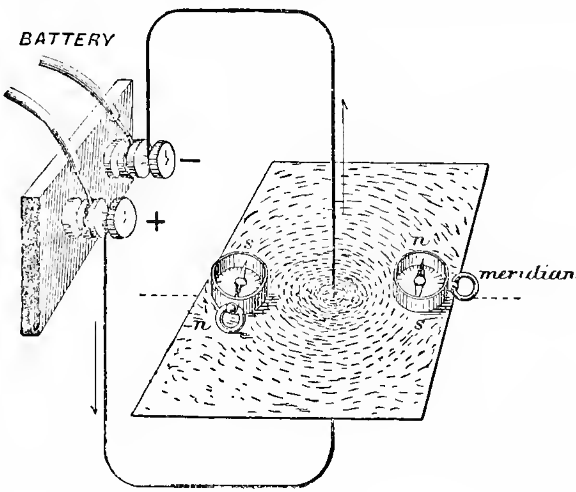


Fig. 124.

in dealing with this subject is that discovered by Oersted 70 years ago, viz., the magnetic character of and effects produced by currents of electricity. A simple experiment is sufficient to exhibit the magnetic field set up in the neighbourhood of a conductor carrying an electric current. A piece of stout copper wire, say No. 14, is pierced through the centre of a sheet of stiff cardboard (Fig. 124), and carried vertically for two

or three feet before it is bent round to the terminals leading to a battery or other source of current. The card should be supported uniformly underneath, so as to present a flat, rigid surface, and the battery should consist of a few cells of large plate area, such as secondary batteries, in order to furnish a strong current. If the latter are used there should be an adjustable resistance between the battery and the experimental wire, which can be gradually reduced until the current is of sufficient strength to exhibit the effects, otherwise the experimental wire, being of neligible resistance, short-circuits the cells. Iron filings sprinkled over the card while the current is passing arrange themselves in circles round the wire. This exhibits the form of the magnetic field surrounding a straight conductor conveying a current, and demonstrates the magnetic character and properties of the current. What most concerns us in the use of this principle is the measurement of the field; in other words, we must be able to calculate the intensity and direction of the field at any point in the vicinity of the conductor. The direction of flow of the circular lines of force according to the usual assumption (para. 104) may be readily observed by exploring the same with a small magnetic compass-needle as shown in the figure. Suppose a line drawn across the card in the direction of the magnetic meridian, and two compass-needles placed on the line, one on each side of the conductor. Before the current is switched on to the wire the two needles will point along the meridian line, both, of course, with their north poles pointing towards the north. Now, if we connect the battery to the terminals, so as to send a current *upwards* through the wire, we shall observe on switching on the current that the needles move sharply into positions nearly at right angles to the meridian line, each pointing the opposite way. The north pole of the one on the right-hand side of the wire will point *away* from the observer, and the same pole of the other needle will point *towards* the observer, as shown. And if we move one compass needle round the wire, starting from one point and coming back to the same again, we shall observe that the needle turns on its pivot through a complete revolution—in other

words, it shows the direction of flow of the curves of force to be continuous. According to the assumption that the north pole of a needle points to the direction of flow of the lines (para. 104), we find that, in this case, the flow is round the wire in the direction in which one would unscrew a screw (*a*, Fig. 125). To remember the direction, perhaps the screw is the best analogy, for in unscrewing a screw we are moving it bodily upwards, *i.e.*, in the same direction as the current flows in the wire. The converse case (*b*, Fig. 125) is with a descending current in the wire, the direction of flow of lines being reversed, and coinciding with the direction of turning a screw when screwing it downwards. Again, suppose we are using a spanner to screw

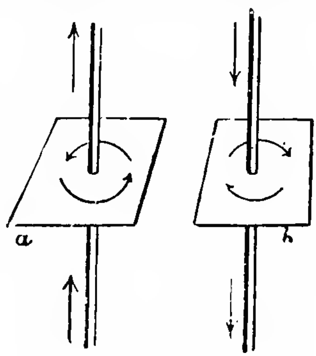


Fig. 125.

a nut off or on a bolt, and we take the bolt to represent the conductor, and the direction in which the nut moves *bodily* along the bolt to represent the direction of the current in the conductor, then whichever way the bolt is placed, or in whatever direction the nut is moved, the direction of the circular lines of force round the conductor will always be the same as that in which the spanner is turned. The student will find it quite easy to recollect the direction of field for a given current by these practical analogies. In moving the needle round the wire it does not quite set itself as a tangent to the circle of force, because it is also affected by the earth's horizontal field,

but with the strong current through the wire necessary to exhibit the circles of iron filings the earth's field has, in comparison to the circular field, very slight influence on the needle. It is this principle which enables us to find the direction of flow of a current in a conductor by the direction in which a compass needle, placed near it, sets itself (para. 66). When a force acts along the circumference of a circle the *direction* of the force at any given point in the circumference is a tangent to the circle at that point, and this tangent is a line at right angles to the radius of the circle drawn from the centre to the point. If, therefore, we consider any one of the circles of force surrounding a conductor conveying a current, it is clear that, whatever point we choose in that circle, the magnetic force is acting there at right angles to the radius of the circle (drawn from the conductor to the point) and at a tangent to the curve. Further, if we consider two straight conductors side by side (Fig. 125), in which equal currents are flowing in opposite directions, it is evident that those portions of the circles of force which are between the conductors coincide in direction. The magnetic field at a point equidistant from both conductors, and in the same plane, is therefore twice the intensity of that which would exist if one of the conductors were removed—in other words, there would be twice the force acting on a magnetic needle placed at the point. But a still greater intensity of field may be produced at the point by bending the conductor into the form of a circle (Fig. 127), of which the given point is the centre. Here all the circles of force act in the most direct and efficient way, so as to add up their effects in one direction. This direction, by what has been said, will be understood to be entering the ring *from* the observer, the current being in the direction indicated by the arrows in the figure. The relative strengths of field produced (1) by a ring whose centre is the point p, and (2) by two very long straight conductors carrying the same strength of current, each of which is separated from the point by a distance equal to the radius of the ring (the conductors and the point being in the same plane), is in the proportion of 2π to 4, or as $6{\cdot}28 : 4$.

The same physical effect of multiplying the magnetic action by carrying the conductor completely round the point where

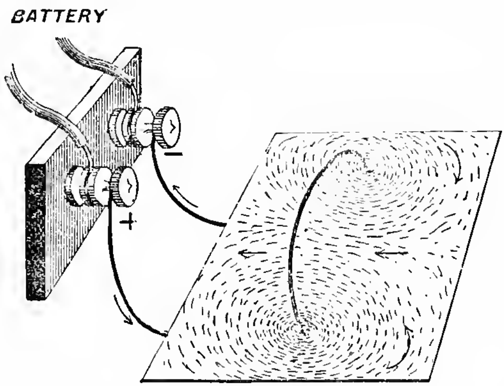


FIG. 126.

the field is required, is obtained in the same way, whether the conductor is in a circular form or otherwise. We shall,

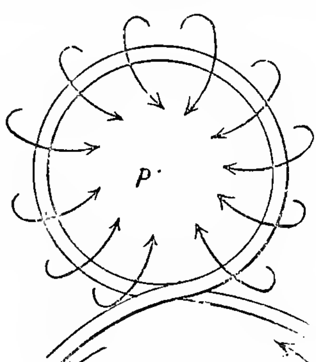


FIG. 127.

however, confine ourselves to the circular form of conductor or coil, this being the general form for standard galvano-

meters, and that which is the best for quantitively determining the magnetic field due to a current. The appearance of the lines of force, as shown by iron filings when the conductor is bent into a semi-circle, is shewn in Fig. 126, where the direction of the current and lines of force are indicated. The next step will be to point out the law by which the intensity of field in C.-G.-S. units due to a current may be determined, and to instance its application to solenoids and galvanometer coils.

144. Direction of Magnetic Field in the Region of a Circular Current.—Only a few months back the attention of the scientific world was directed once more to the life and labours of that great French mathematician and physicist, André Marie Ampère, the occasion being the unveiling of a statue in his memory erected by his fellow citizens in the city of Lyons. Immediately upon the news reaching Paris of Oersted's discovery of the action of a current of electricity upon a magnetic needle towards the close of 1820, Ampère keenly followed up the subject, and very soon after gave to the world his electro-dynamic theory, embracing the laws governing the mutual action between conductors carrying currents of electricity, based upon which we now have some remarkable current and electricity meters. Among these may be cited Siemens' electro-dynamometer, in which a current circulating in two coils, one fixed and the other movable, is measured by the amount of mechanical force required to counteract their mutual attraction. Again, the Ferranti meter, in which the current to be measured passes radially through a mercury bath, and then through a fixed coil outside it, causing rotation of the mercury, and setting in motion a train of clockwork indicating the quantity of electricity passed through the meter in a given time; and more recently the beautiful ampere balances of Sir William Thomson, in which the movable coils are at each end of a balance arm, and are attracted, one up and the other down, by a pair of fixed rings, embracing each movable coil. In this instrument a sliding weight is moved along the bar until a position is found, such

that the balance arm is restored to equilibrium, and this position indicates on a scale the strength of current passing through the coils. Measuring instruments such as these, designed on the principle of the attraction between fixed and movable portions of an electric circuit, have the invaluable property of being capable of measuring the strength of *alternating* as well as direct currents.

But, further, Ampère gave a precise explanation and theory of the magnetic character of electrical circuits; in fact, he showed that when a conductor carrying an electric current is bent into the form of a single ring or a series of rings side by side, such as a spirally-wound coil, that the behaviour of such is identical with the behaviour of a magnet. The spirally-wound coil, in which a current is circulating, acts, as regards attraction, repulsion, and external magnetic effects, exactly like a cylindrical bar magnet, presenting, as it does, at its extremities north and south magnetic poles. Similarly, a ring-shaped coil, through which a current circulates, presents opposite magnetic poles at its two faces, being, in fact, identical in magnetic effect to a thin slice off a cylindrical bar magnet. Such a thin slice or disc of magnetised iron is called a *magnetic shell*, and the ring-shaped coil carrying a current which is equal in magnetic action to such a shell is termed its *equivalent* magnetic shell. Further, every plane closed circuit of whatever external shape can be shown to have its equivalent magnetic shell.

It is with this part of the subject that we have now more particularly to deal, and to make the subject as easily understood as possible we shall first consider only the *direction* of the magnetic field in the neighbourhood of a circular current, and then show how its exact value at different points may be determined by Ampère's laws. Suppose, first, a portion of a conductor carrying a current, bent into a ring as in Fig. 128, the arrows representing the direction of the current. What is most useful to know with reference to the action of galvanometer coils is the direction and magnitude of the magnetic field along what is called the axis of the coil. The axis of a coil is an imaginary line in the same position with regard to a coil

as a shaft is in with regard to a pulley keyed to it—that is, it is a line (such as the line A B in the figure) whose direction is at right angles to the plane of the coil, and which passes through its centre. Lines of force will enclose the conductor

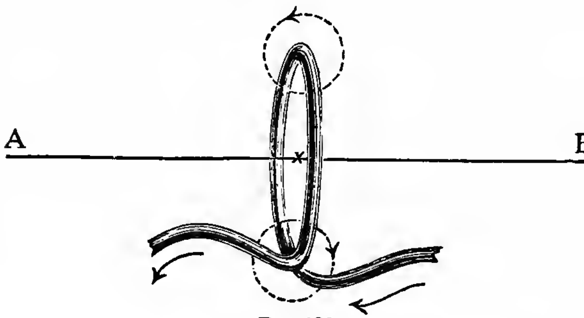


Fig. 128.

in circles at every point throughout its length. Now let us consider separately for a moment those circles of force which lie in the same vertical sectional plane of the ring, one at the

Fig. 129.

top and the other at the bottom, and let us examine the magnetic field they create along the axis A B. Having found this we can add up the fields set up by every sectional plane of the ring, for every one of these planes acts alike. The direction

of the circles of force will be understood to be as indicated in the figure, because the current is flowing *towards* us in the top of the ring, and *away* from us in the lower portion. Take, first, the action of the upper portion of the ring and let us consider it separately, the section of the wire being represented at W (Fig. 129). On the axis A B let us mark out a series of points (1 at the centre and 2, 3, 4 on each side of it) at which to determine the field set up. Circles of force emanating from the conductor W as centre will pass through all these points. Now we have already stated that when a force acts along a circle, its direction at any given point is at a tangent to the circle.

FIG. 130.

We have familiar examples of this tangential action in centrifugal separators. For example, if the dotted circle in Fig. 130 represents a body revolving in the direction shown, the forces acting at any moment when it passes the points A B, C D, are tangential to the circle, or at right angles to the radii at those points, and this is the case for *any* point we may take on the circle. Applying this principle to the circles of force round the conductor, we find the *direction* of the magnetic forces by drawing lines at *right angles* to the radii of the circles passing through the various points, and assigning to them a direction agreeing with the direction of the circles of force. It will be

noticed, then, at the point 1, which is at the centre of the ring, the direction of force is along the axis of the ring, but as we get further away from the plane of the ring the forces become more and more nearly at right angles to the axis. Now,

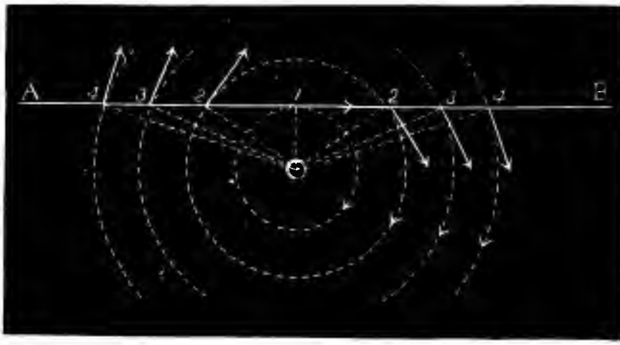


FIG. 131.

consider the action of the circles of force emanating from the lower portion of the ring. Fig. 131 represents this action, where 1, 2, 3, 4 are the same points as chosen above, and the circles of force are in the opposite direction. Proceeding in

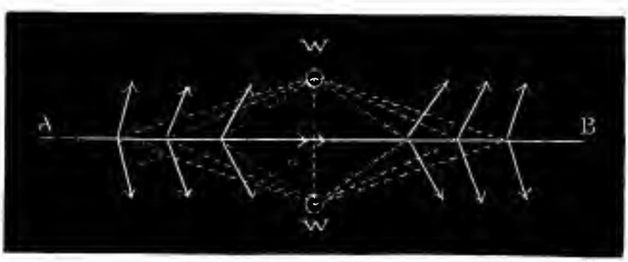


FIG. 132.

the same way, we find that at the point 1 (the centre of the ring) the force acts along the axis of the coil *in the same direction* as the force due to the upper portion of the ring, and the forces at the other points act at the same angles relatively to the axis as those above. Now combine the two sets of forces by

setting them out together along the axis (Fig. 132), the upper and lower sections of the conductor composing the ring being indicated at W W. Here we find that although the forces diverge more and more from the axis as we proceed further away from the plane of the coil, yet each pair of forces has its *resultant along the axis*, and all resultants are in the same direction as the two forces at the centre due to both portions of the conductor. Further, the resultants of these pairs of forces become less and less as we take them further from the plane of the coil, because the forces themselves act more and more in a direction *opposing* each other, and therefore tend to reduce the intensity of the horizontal resultant. By reversing the current in the coil the whole of these forces would also be reversed in direction.

145. Intensity of Field in the Region of a Circular Current. The C.-G.-S. Unit of Current.—In the preceding paragraph we investigated the direction of field along the axis of a ring conductor carrying a current, and that direction we considered in one plane only of the ring. What is true for one plane is true for all sectional planes of the ring, and a little consideration will show that the *entire* magnetic field (or force on unit pole) set up at a point on the axis, that is, the field due to the current in the ring conductor, *as a whole*, will be acting at the point round a cone-like surface. This effect of the entire field at the point may be understood by the perspective view of the ring and the forces set up by it at the point p in Fig. 133. Here A B is the axis of the ring as before, and if we imagine a unit north pole at the point p, it would be urged outwards from the ring along the axis towards B. In this direction there must evidently be one resultant force equal to the combined forces acting round the cone at the point, and it is this resultant that we have to find. Now, any point on the axis A B is equidistant from every portion of the ring conductor; and wheu this is the case the *total* intensity of field at the point is directly proportional, by Ampère's law, to the length of conductor multiplied by the strength of current, and inversely proportional to the square of the distance between

the point and conductor. The distance between any point on the axis and the conductor is simply the common radius to all those circles of force which pass through the point. If we take R as this radius, C as the current, and L as the length of conductor composing the ring, the total intensity of field at the point is proportional to

$$\frac{C\,L}{R^2}.$$

Now, to find the resultant of this field along the axis, let us consider one plane through the cone, as before. Here we have two diametrically opposite forces, each, say, equal to F (Fig. 134), acting at the point p, as tangents to the circles

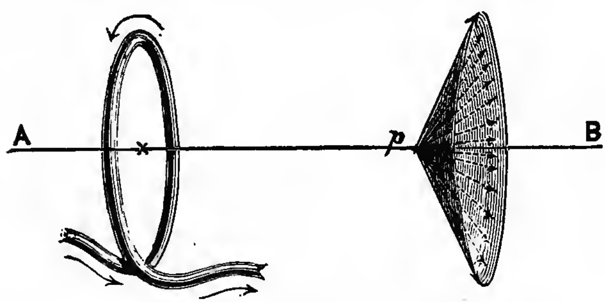


FIG. 133.

of force through the point, or in other words, at right angles to the common radius R, of the circles of force through p. Completing the parallelogram of these forces, and calling f the resultant of the pair, we find by similar triangles that

$$f : F :: 2\,r : R,$$

where r is the radius of the ring conductor whose opposite sectional portions are at W W, that is,

$$f = F\,\frac{2\,r}{R}.$$

Now it is evident that the resultant of the cone of forces bears the same relation to the combined intensity of those

forces as the resultant (f) of the pair (2 F) bears to that pair. Therefore we have the relation,

$$\frac{\text{Resultant field}}{\text{Total field}} = \frac{f}{2\,F}.$$

Substituting the value of f as obtained above, and cancelling, we have

$$\text{Resultant} = \text{total field} \times \frac{r}{R},$$

$$= \frac{C\,L}{R^2} \times \frac{r}{R}.$$

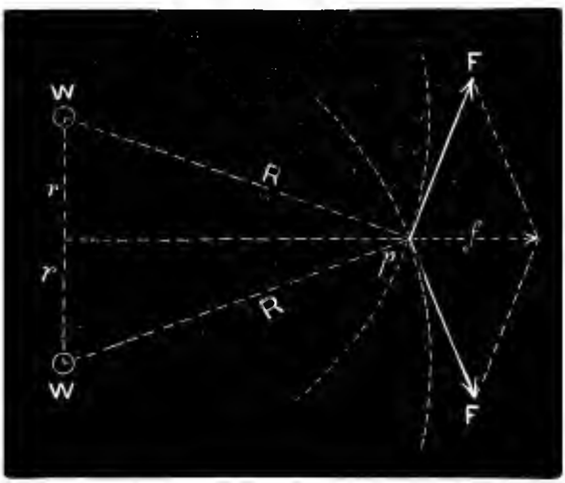


FIG. 134.

Now, the length of conductor (L) in a ring of radius r is equal to $2\,\pi\,r$, and therefore we have,

$$\text{Resultant along axis} = \frac{C\,2\,\pi\,r}{R^2} \times \frac{r}{R},$$

$$= \frac{2\,\pi\,C\,r^2}{R^3}.$$

This is a very important expression, as it is proportional to the field at any point along the axis of a coil. We have not

yet taken the quantities in any fixed system of units, so that the expression as it stands must be regarded as simply proportional to the field.

Now in this expression we notice that the numerator is constant for a given current flowing in a ring of given size, and the field at any point along the axis of the ring is inversely proportional to the cube of the radius of the circles of force through that point—that is, the cube of the distance of the point from the conductor.

It is interesting to represent graphically the rate at which this field decreases as we take points further and further from

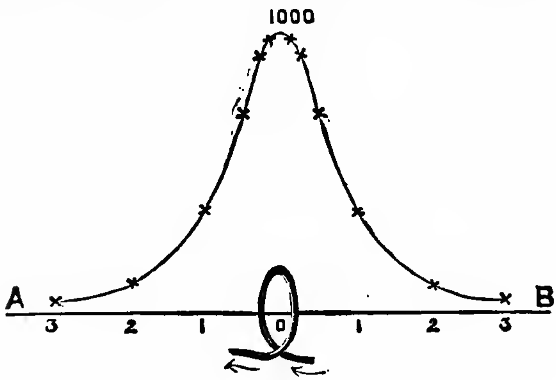


FIG. 135.

the coil. All that is necessary is to take measured distances along the axis A B of the coil (Fig. 135), starting from 0 at the centre of the coil, and for each of these distances calculate the radius R, then cube it and take its reciprocal, the resulting number being proportional to the strength of field at the points taken. For instance, taking the radius of the ring as unity, and the strength of field at its centre as 1,000, we find that at the distances one, two, and three the field is proportional to 350, 89, and 32 respectively. A few more points nearer still to the coil should be worked out, as the curve slopes more gradually at very short distances from the coil.

For example, at distances $\frac{1}{8}$, $\frac{1}{4}$, and $\frac{1}{2}$ from the coil the field is proportional to 977, 913, and 715 respectively. These six points on the axis of the coil are sufficient to show the general rate of decrease of field along the axis. It will be noticed in Fig. 133 that R forms the hypothenuse of a right-angled triangle, of which one side is the radius (r) of the ring, and the other is the distance (d) of the point on the axis from the centre of the ring. We have, therefore, the relation (Euc. I., 47)

$$\mathrm{R}^2 = r^2 + d^2$$

whence

$$\mathrm{R}^3 = (r^2 + d^2)^{\frac{3}{2}},$$

from which the cube of the radius R may be found for any distance from the centre of the ring. Raising vertical ordinates proportional in height to the field at the several points chosen, and connecting them, we obtain the curve in Fig. 135, showing the rate of decrease of field on each side of the ring along its axis.

It will be noticed in working out the above values that the field at every point we take depends on two things, viz., the radius of the coil itself, and the distance away from the coil along the axis, and it will be found that whatever size coil we consider (*i.e.*, of whatever radius), and whatever current flows through it, the field will always decrease gradually (as in the above curve) at short distances from the coil on either side, then suddenly the fall will be rapid, and then gradual again. And the fall in strength of field is always most rapid at a distance away from the coil equal to half its radius. We might, therefore, call this the critical point on the axis, because a little further away than this point the field is considerably feebler, and a little nearer to the coil than this point the field is considerably stronger. Returning to the first part of this paragraph, it will be seen that by Ampère's law the field at the *centre* of a coil is proportional to

$$\frac{\mathrm{CL}}{r^2},$$

because the radius (R) of the curve of force through the centre is now equal to the radius (r) of the ring. And of these three

quantities, viz., field, current (C), and length (L and r^2), which are related to each other in the above proportion, there are two, viz., field and length, of which we have already defined units; it only remains to define unit current. Now, if we take a coil of unit radius (r), and consider only the magnetic field at the centre *due to unit length* (L) *of the wire* (*i.e.*, any part of the coil equal in length to the radius), and, further, suppose that a certain current flows in the coil which causes the field at the centre (due to unit length of wire) to be equal to unity, then the only value we can give the current on the basis of an absolute system of units is to take that current as the unit current. Supplying, then, the units on the C.-G.-S. system, we can at once define the *absolute* unit of current as that current which, flowing in a ring conductor of 1 centimetre radius, produces at the centre of the ring a magnetic field equal in intensity to 1 C.-G.-S. unit *for every centimetre length of wire in the ring.* Further, since the ring must be 2π centimetres in circumference, the *total* intensity of field at the centre is equal to 2π C.-G.-S. units. The practical unit of current, to which is assigned the name of Ampère, is chosen as equal in value to *one-tenth* of the absolute unit, and therefore one ampere flowing in a ring conductor of one centimetre radius produces a field of

$$\frac{2\,\pi}{10}, \text{ or } \frac{\pi}{5} \text{ C.-G.-S. units.}$$

If the ring or coil consists of more turns than one, say n turns, the field is increased n times, and the complete expression for the field at any point on the axis becomes

$$\frac{2\,\pi\,\mathrm{C}\,n\,r^2}{\mathrm{R}^3} \text{ C.-G.-S. units,}$$

which at the centre of the coil (where $r=\mathrm{R}$) becomes

$$\frac{2\pi\,\mathrm{C}n}{r} \text{ C.-G.-S. units,}$$

where r is then the *mean* radius of the coil in centimetres.

Example.—The coil in Sir Wm. Thomson's graded current galvanometer (described para. 132) consists of six complete turns of stout copper strip, the inside diameter being 6 centi-

metres and the outside 10 centimetres. What is the intensity of field produced by a current of 30 amperes flowing through the coil, at a distance of 10 centimetres from the centre of the coil along its axis, and what is the field produced at the centre of the coil?

Mean radius of coil (r) $=4$
Current in C.-G.-S. units (C) $=3$
Number of turns (n) $=6$
$2\pi C n r^2 = 1809{\cdot}5$
log of 1809·5 $= 3{\cdot}25755$

$$R^3 = (r^2+d^2)^{\frac{3}{2}} = (16+100)^{\frac{3}{2}}$$

log of 116 $= 2{\cdot}06445$

$$\text{log of } R^3 = \frac{2{\cdot}06445 \times 3}{2} = 3{\cdot}09667$$

Subtraction gives log of answer $= {\cdot}16088$

Therefore,

Intensity of field $= 1{\cdot}448$ C.-G.-S. units.

$$\text{Field at coil centre} = \frac{2\pi C n}{r} \text{ C.-G.-S. units.}$$

$$= \frac{113}{4} = 28{\cdot}25 \text{ C.-G.-S. units.}$$

146. Magnetic Fields due to a Coil and Magnet Superposed.—We have already discussed the case of two uniform magnetic fields superposed at right angles to each other (para. 119). Referring again to the figure (Fig. 80), it was shown that the ratio between the two superposed fields f and H was the tangent of the angle which H, the total controlling field, makes with the resultant. Let us now take the practical case which occurs in every tangent galvanometer, viz., the superposition at right angles to each other of the controlling field due to the earth or a permanent magnet, and the field due to the current in the coil. A little experiment carried out with a Thomson potential galvanometer coil and a battery of secondary cells may serve to make this action clear. This instrument is of the same external appearance as the current galvanometer described in para. 132, but instead of a

few turns of stout copper strip the coil is wound with some 8,000 turns of German silver wire, amounting to a resistance of 9,830 ohms. The needle-box, movable along the wooden platform, being removed, the magnetic fields produced in the region of the coil were examined by iron filings sprinkled on a sheet of stiff card placed on the platform. First, without any current through the coil, the controlling magnet N S was placed over it in the usual position as used with the instrument (Fig. 136). The iron filings then revealed the

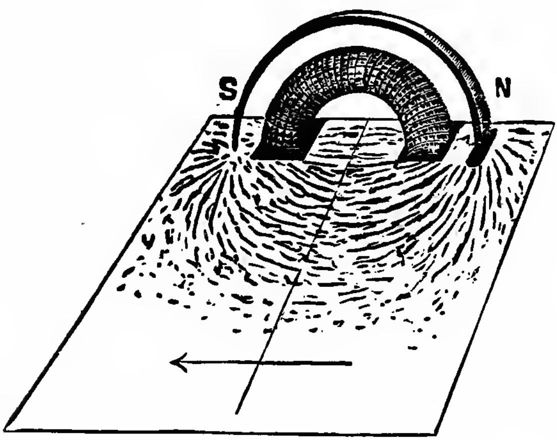


FIG. 136.

kind of field produced in the region of the coil. The lines of force here pass straight from one pole of the magnet to the other, producing at the centre of the coil a field parallel to its plane. When in use the coil and magnet are put in the plane of the magnetic meridian so that the earth's horizontal field is added to that of the magnet. The earth's field alone is far too weak to be exhibited by iron filings. The central line in the figure represents the axis of the coil, and is the line along which readings are taken with the instrument. Along this axis we may consider the controlling field produced by the magnet and

the earth to be everywhere in the direction of the arrow, viz., at right angles to the axis. For even at positions along the axis remote from the coil where the lines are in curves, the direction of each line of force *as it crosses the axis* is a tangent to the curve, and, therefore, at right angles to the axis. Secondly, the controlling magnet was removed, and the ends of the coil connected by leading wires to the terminals of 21 cells, giving a difference of potential of 42 volts at the terminals of the coil. A vigorous effect was then produced on the filings, which arranged themselves into the form in Fig. 137. It will be noticed that as regards the axis of the

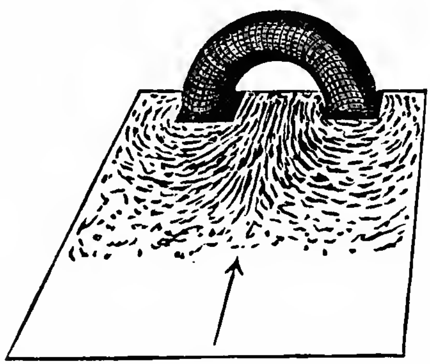


FIG. 137.

coil, this field is at right angles to that set up by the controlling magnet. The current was sent round the coil in such a direction as to cause the direction of the lines to be *towards* the coil, as shown by the arrow. Lastly, the controlling magnet was placed over the coil, as at first, the current being kept on the while, resulting in the change in direction of the filings represented in Fig. 138.

Here we have an actual representation of the resultant field, that is, the character of the field resulting from the combined effect of two distinct fields at right angles to one another. The arrow in this figure indicates the direction of the lines of the

resultant field at the centre of the coil. As we take points on the axis further away from the coil the effect of the latter becomes rapidly decreased, and the controlling field predominates, so that the lines become nearer and nearer at right angles to the axis. The direction of this resultant field is precisely that taken up by a magnetic needle free to move when placed at a given position on the axis of the coil. If the needle is at the centre of the coil we have an ordinary tangent galvanometer, and if the box containing the needle can be shifted along the axis of the

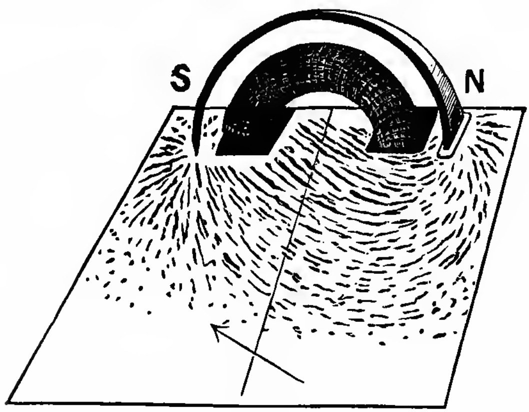


FIG. 138.

coil, as in Sir William Thomson's graded galvanometers, we have instruments which still follow the tangent law, and which have, in addition, the advantage of great variation in sensibility, as before explained. Let us form a practical idea of the strengths of the two fields at the centre of the coil, their resultant, and its direction in the above case. The outside diameter of the coil was 14 centimetres, and the inside 6 centimetres; the mean radius was therefore 5 centimetres. The current was equal to

$$\frac{\text{volts}}{\text{ohms}} = \frac{42}{9830} = \cdot 00427 \text{ amperes.}$$

Dividing this by 10 we have the

$$\text{current in C.-G.-S. units} = \cdot 000427.$$

The number of turns of wire being 8,000, the field at the centre of the coil due to the current was :—

$$\frac{2\pi \times \cdot 000427 \times 8000}{5} = 4 \cdot 3 \text{ C.-G.-S units.}$$

Now, the controlling field at right angles to the above consisted of the field of the magnet, which was 10·56 C.-G.-S. units, plus the earth's horizontal field ·18 unit, total 10·74 units. The ratio of the coil field to the controlling field is the tangent of the angle which the resultant makes with the controlling field, that is

$$\frac{4 \cdot 3}{10 \cdot 74} = \cdot 4003 = \text{tangent of } 22^\circ \text{ nearly.}$$

A small magnetic needle freely suspended at the centre of the coil would therefore turn through this angle with the above potential difference applied at the terminals of the coil.

147. General Character of Field Due to a Solenoid.—In considering the way to determine the strength of magnetic field at any point on the axis of a ring conductor carrying a current, we gave ourselves a more extended and more general view of the subject than when determining the field at the centre of the coil only. In other words, the latter case is involved in the former, and the student will find it best to master the more general case, because he will then be able to apply it to any special case required. The determination of the field at any point along the axis of a solenoid is the most general case of all, and it will therefore be the last case we shall examine in this section. At the same time, this most general case is the most difficult one to be followed and understood, and the writer hopes, at any rate, to lead up to the final result by a series of easy steps. It will be well here, as previously done, to get a general idea of the character of field exhibited by a solenoid; next, to consider the *direction* of the field, and finally its *intensity* in C.-G.-S. measure at any point along the axis. This being done, the

student will find that he is in a position to intelligently understand the various forms in which galvanometers are constructed. A single low-resistance cell connected to the terminals of a solenoid wound with a few layers of No. 18 or No. 16 double cotton-covered wire will be sufficient to exhibit the general contour of the magnetic field set up in and around the coil. It is better to mount the solenoid on a bit of wood, as in Fig. 139, and carry the two ends of the coil through the wood and along two slots underneath leading to their terminals. Done in this way you can have two sensible terminals to connect your wires to, instead of the flimsy little terminals sometimes screwed on to

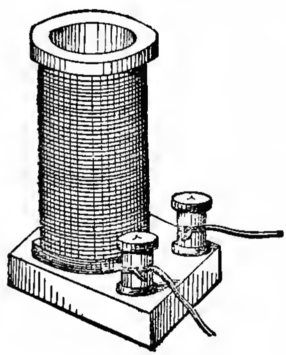


FIG. 139.

the cheek of the bobbin, which frequently cause splitting of the same, or, at all events, make bad contact with the coil. After winding, the bobbin may be fixed rigidly to the wooden base by countersinking a hole in the latter deep enough to receive the lower cheek of the bobbin flush with the level of the base. A couple of semi-circular pieces of brass plate covering the cheek and the contiguous part of the base may then be screwed down to the latter, holding the coil firmly in position. Or a piece of brass sheeting may be cut so as to fit over the top cheek and come down in a half-inch strip each side, and be firmly screwed on to the base below. In any case,

screws should not be fastened into the cheeks themselves. The solenoid can be fixed horizontally in much the same way on to a base board, but its position will depend on what kind of experiments are to be made with it. When finished, it can be used for a variety of experiments, such as the mechanical pull it exerts upon an iron core to draw it into the coil. This can be varied by trying different kinds of core, such as, for instance, a core built of a bundle of small iron

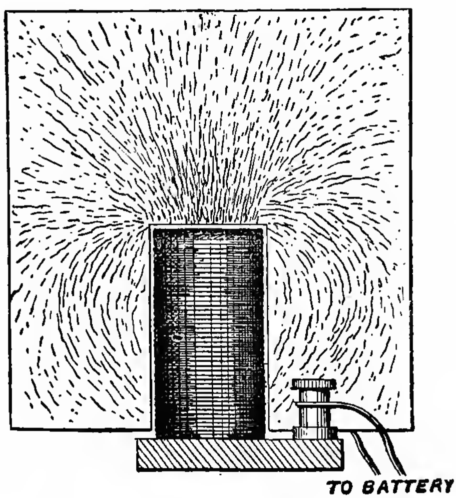


Fig. 140.

wires and cores of different shapes with pointed and blunt ends, &c., measuring the pull on a spring balance, when, say, a given core has different portions of its length introduced into the coil. The outside layer of wire on the coil should be well brushed over with thick shellac varnish (shellac dissolved in alcohol). This soaks in and hardens the cotton covering, protecting it from abrasion in handling. Curves made from the plotting of the observations above indicated are very useful in showing the action of cores working in solenoids in arc lamps,

especially the difference between the pull on a core which moves *with* the carbon rod, as, for instance, in the Pilsen lamp, and that on a core which generally maintains a certain position relative to the coil, as in the Brush or Siemens. The foregoing remarks are merely to indicate what useful experiments and tests may be carried out with a solenoid; to go into detail in this direction would more befit a separate treatise on electric lighting, and is outside our present limits. Taking, then, such

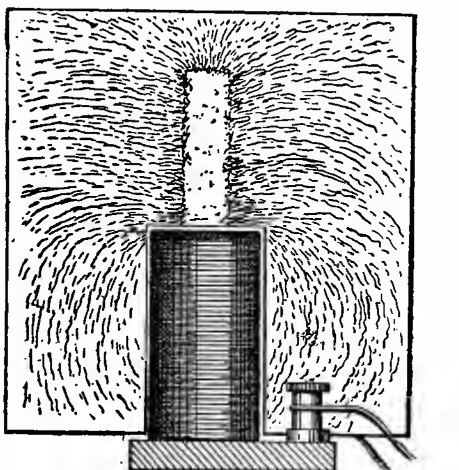


FIG. 141.

a solenoid and laying it down horizontally, we can examine the character of field by iron filings sprinkled on a card supported close to it, as shown in Fig. 140. When the current is on and the filings are sieved down gently they will be found to arrange themselves as indicated in the figure, and it is easy to see that the solenoid exhibits the same kind of field as a permanent magnet whose length is in the same direction as the length of the solenoid. Further, we can better understand what goes on in the interior of a bar magnet by noticing

the kind of field that we get *inside* the solenoid. This is best examined with a small compass needle, which we shall do shortly. Taking an iron core, fitting into the solenoid, and of about the same length, it is interesting to observe that when the core is very nearly out of the coil, as in Fig. 141, the shape of the field is considerably altered. The reason that no iron filings adhere to the card just over the iron core is because the core itself absorbs those lines of force which otherwise would

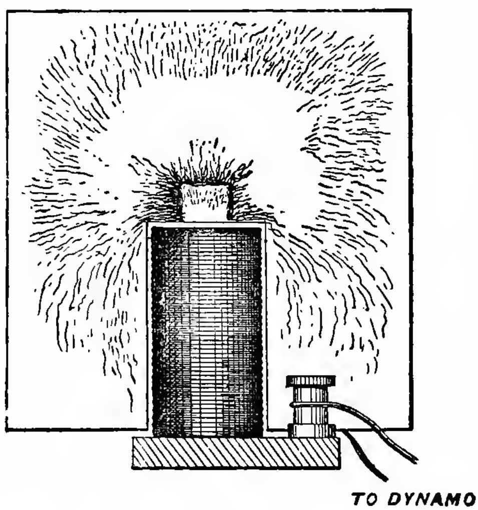


FIG. 142.

pass through the air and attract the filings. So easy a path does the iron core offer in comparison to the air that the lines pass straight from the interior of the coil into the iron core for some distance before passing outward in the usual curves to complete the magnetic circuit, and this causes the alteration in the shape of the field. The projecting end of the core is strongly magnetised, and its magnetism will be reversed if the direction of current in the coil is reversed. Also, if the core is right inside the solenoid, we have virtually an electro-magnet,

and with a direct current we get the same field as we have before shown for electro-magnets. With an alternating current through the coil, however, such a vibration is set up that immediately the filings touch the card the greater part of them dart at once to the core. This effect is shown in Fig. 142.

148. Direction of Field Due to a Solenoid.—A solenoid, mounted as in Fig. 143, is very suitable for observing the direction of field set up. One layer of stout wire, say, No. 12, is first wound round a cylindrical surface such as a battery jar, to

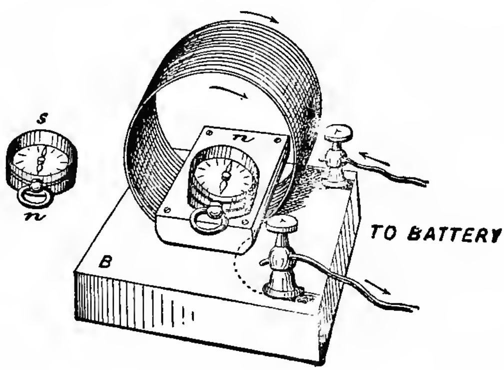


FIG. 143.

give it the proper shape which it will be found soft enough to retain itself after slipping off. It is better to leave a little clear space between each turn, so that afterwards when experimenting you can look down into the middle of the coil. If this is done there is no need to have the wire insulated. Let the two ends be passed through the wooden base block B carrying the two terminals, and solder the wires underneath this block to the extremities of the terminals. Clamp the coil down by a flat piece of wood screwed down to the base block, as shown. This keeps the solenoid firmly in position, and affords a little table whereon to put your compass needle when examining the

field in the interior. A convenient size of solenoid is about from four to six inches in diameter, and, say, one foot long. Before passing the current through let the solenoid be placed so that the direction of its length is at right angles to the magnetic meridian, and then put a compass needle inside, or, better still, place the compass inside first and then turn the whole apparatus bodily round till the needle points at right angles to the direction of length of the solenoid. If another compass needle be placed *outside* the solenoid in the position shown in the figure, both needles will point the same way, north and south, before the current is passed into the coil; but immediately the current is switched on the needles will turn

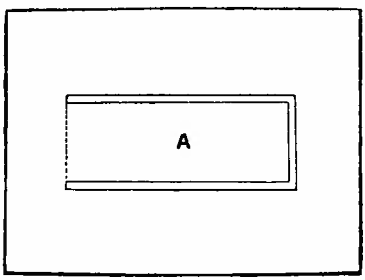


Fig. 144.

sharply in opposite directions, and when at rest will lie very nearly in the direction of length of the solenoid. Notice should be taken of the direction of the current and the direction of movement of the needles. In the figure the needles are shown as they are deflected when the current passes through the solenoid in the direction of the arrows. We now see the direction of the field by the direction towards which the north poles of the needles point, and we find by exploring all round the solenoid that the lines seem to flow along the interior space of the coil and then to separate into two streams passing *backwards* along the exterior sides of the coil, and then turning in again to the centre. This can be

best observed by cutting out a piece of card as in Fig. 144, bending it upwards at the dotted line, and slipping it over the coil in the manner shown in Fig. 145, the centre portion, A, being then bent downwards again so as to lie along the interior of the coil. By passing the current through the coil again, and letting fine iron filings fall on the card, the character of the entire field is delineated. Supposing the current in the coil to be in the same direction as in Fig. 143, we shall find that wherever we place small compass needles the direction in which their north poles turn is along the arrows marked on

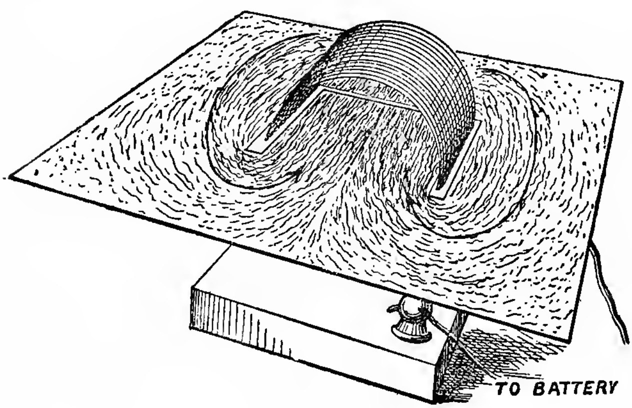


FIG 145.

the card. Now the card only exhibits the field in *one* of the sectional planes of the coil, but it is obvious that the field is the same for all sectional planes; and, hence, when we think of the field of a coil or solenoid we must carry in our minds the conception that the field entirely envelopes the coil. In other words, the lines of force fully occupy the interior space of the coil, and after passing through this space divide evenly and flow *backwards* round the outside; and with this conception it will be easily understood that if we attach two magnetic needles rigidly together, as in Fig. 146,

with their north poles pointing opposite ways, and freely suspend the two by a silk thread, so that one is inside the coil and the other outside, as shown, both needles will turn in the same direction when a current is sent through the coil. The two needles together, or, as the combination is technically called, the "system of needles," is hardly at all affected by the earth's horizontal force, because what force it exerts on one needle is counteracted very nearly by the force on the other; in fact, if the two needles were of exactly the same magnetic moment, the "system," when allowed to swing freely in the earth's field, would not come to rest in any definite position,

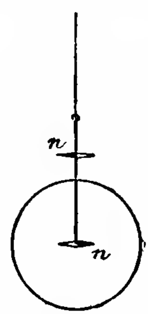


FIG. 146.

and hence the "system" is called an "astatic" system. It will be seen that as the earth's field has very little controlling effect on it (only just enough to keep it at zero) the astatic system is very sensitive. It is usual to have two separate coils, one above the other, and each containing and acting upon one of the needles, but when the idea was first developed one coil only was used. Faraday mentions in his renowned "Experimental Researches" the use he made of an astatic system with one coil (Fig. 146) in carrying out some induction experiments. And the writer has introduced it here believing it to be of interest in connection with the direction of field in and around one coil *alone*; and, indeed, in an earlier part of this series

(para. 57) reference was made to the subject, the explanation of which now will make the case given there better understood.

Lastly, if a compass needle placed inside the coil be moved outwards along the axis either way, it will be found that its deflection keeps pretty nearly in line with the axis until it emerges from the coil, when the deflection rapidly falls off; and ultimately, by removing the needle further and further away, we find the field of the coil exerts practically no more force on it, and it rests in the meridian once more. It is precisely this variation of field along the axis that we wish to determine for any kind of solenoid; and further than this, to determine in C.-G.-S. units the exact intensity of field at any point.

149. Intensity of Field at the Centre of a Solenoid.—In the treatment of this subject there is involved the use of a branch of mathematics not frequently understood either by those who are commencing their electrical studies or even by old and tried practitioners. But while it is very serviceable to be able to sum up a series of extremely small quantities by the usual mathematical process, it is very much more important to have a common-sense view of what has to be done to attain the result aimed at. It is a cause of disappointment and dismay to many students who have been carefully following up a subject to find themselves, after much labour spent in trying to understand it, handicapped by a mathematical process which is said to attain the desired result, but of which they do not comprehend the meaning. The said mathematical treatment would be quite out of place in these Letters, and if the result were put down accordingly without any explanation of the process by which it is arrived at, the reader would be no better off than before. Hence, it seems desirable to steer a middle course between these two extremes. But the task so proposed has in it somewhat of the difficulty experienced by Ulysses on his voyage from Troy, when, to save his men from the grasp of Scylla, he ran the risk of getting wrecked in the whirlpools of Charybdis; and it is possible that in trying to avoid one alternative we may in some way run foul of the other. The student will find, however, that if he gets a common-

sense view even of the *manner* of arriving at the result, or clears up any previously foggy ideas on the subject, he will certainly have scored another point to the good.

To fix our ideas, let us suppose a solenoid six centimetres long, wound with five layers of wire, 40 turns to the layer, and therefore making in all 200 complete turns. Let the depth of the winding—that is, the thickness of the five layers—be half a centimetre, and the reel on which the coil is wound 3·5 centimetres diameter; this makes the *mean* radius (r) of the coil two centimetres. Suppose that a current of three amperes is passed through the wire, it is evident that it circulates 200 times round the bobbin, and the total current passing round the bobbin is therefore $3 \times 200 = 600$ amperes. Although each turn is really in series with the rest, yet if we look at a section lengthwise through the coil the ends of the wires appear like so many separate conductors, each in "parallel" or "multiple arc" with the rest, and the total current is therefore the sum of the currents carried by each conductor, viz., 3×200; or we may regard the winding as equal to $200 \times 3 = 600$ ampere-turns, which may be replaced, without in any way altering the field, by a single turn, occupying the same volume as the 200 turns, and carrying 600 amperes. This is the most convenient, and, at the same time, the most accurate way of looking at the action of a solenoid. We shall therefore think only of the total current circulating uniformly *once* round the bobbin and occupying the same space as the winding. Or, putting the same thing in a general form, to which any figures may be applied, let n be the number of turns of wire on the solenoid, and C the current in C.-G.-S. units passed through the wire. Then we can regard this as a current of Cn C.-G.-S. units making one turn round the bobbin. Now it is convenient to consider this single turn or belt of current as divided up into a large number of thin rings of current, because then we can calculate the field produced at the required point by each narrow ring. In short, the expression which was derived in para. 145, viz. :—

$$2 \pi \mathrm{C} \frac{r^2}{\mathrm{R}^3},$$

for finding the field in C.-G.-S. units produced at any point on the axis of a ring of mean radius r centimetres, assumes that the current C is in the exact centre of the wire forming the ring. To make this clear, suppose we take a ring of *wire* and a wide flat *ribbon* of copper, which, when bent round, makes a single turn of cylindrical shape, of the same diameter as the ring. Now, although the same current be passed through the single turn of wire and the single cylindrical turn, the field produced by each at their respective centres is not the same; for in the wire the current is concentrated into a narrow space *immediately surrounding* the point where it is required to determine the field, whereas in the wide cylindrical coil the current is spread over the whole cylinder, and it is therefore only a small portion of the current which flows *immediately over* the centre point. Further, it is obvious that the remaining portions of the current which are flowing round parts of the cylinder further and further away from the centre must have less effect in producing field there than would be the case if the whole current was massed together right over the central point, as in a ring of wire. And therefore we can only apply the above expression to rings of wire or coils of several turns whose *breadth* is very small compared to their *diameter.* But it follows that the field produced by the entire cylindrical coil can be estimated by considering its current as divided up into a large number of very narrow, thin rings, the field due to each of which at the required point can be calculated by the above expression, and the several results for each ring so obtained finally added together. This gives the field produced at any required point on the axis due to the current *in every part* of such a cylinder, and is, in fact, the way in which the total field due to a solenoid is calculated. For the whole coil of wire of a solenoid is cylindrical in shape, and we have seen that it may be considered as one turn occupying the same volume, which is therefore also cylindrical.

The question naturally occurring next is, how many rings are we to consider the cylindrical turn or belt of current divided into? Well, first let us divide it into rings of one

centimetre width. Now if the solenoid is l centimetres long there will be l such rings, and the current (C_1) carried by each will be the total current in the cylindrical belt divided by the length l; that is,

$$\frac{C\,n}{l} = C_1 \text{ in C.-G.-S. units.}$$

This is usually spoken of as the current per unit length of solenoid. We must carry this subdivision of the rings to a still

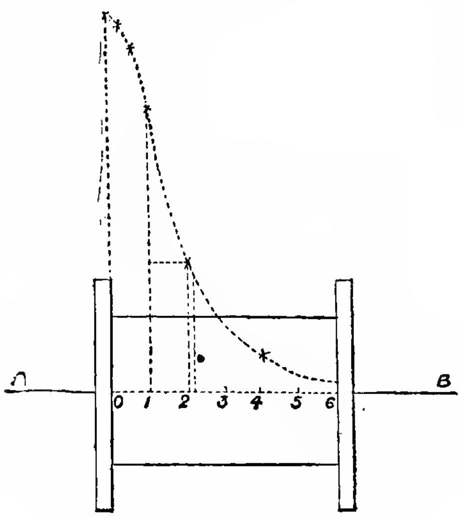


FIG. 148.

greater extent; but this will be best understood by plotting a curve over the solenoid (Fig. 148) showing the relation between strength of field at the point O produced by different current rings, and the distance of these rings from the point. The axis of the solenoid is along the line A B, and it is at the point O on this axis that we will first set ourselves to determine the strength of field due to the entire solenoid. We might at once take this point at the middle of the coil; but it is better, as a

more preliminary step, to consider the field at one end of the coil first. Any point on this axis between the beginning and end of the solenoid will, of course, be the centre of a narrow ring carrying a certain fraction of the current per unit length. It is not necessary to consider now what this fraction may be, for the curve will be of the same shape whatever we take it. In fact, as this fraction of the current would be the same for each narrow ring we take, it may be regarded as constant, and the curve plotted by simply working out the reciprocals of R^3 for a few points on the axis. Raising perpendiculars proportional to the figures so obtained, and joining together the summits of these perpendiculars, we get the outline of the curve as shown. The few points on the axis selected, with corresponding numbers proportional to the field they produce at O, are given below. These few points are quite sufficient to show the general shape of the curve.

Position of ring on axis.	Values of $\frac{1}{R^3}$ proportional to field.
0	·125
$\frac{1}{4}$	·12
$\frac{1}{2}$	·1125
1	·09
2	·0425
4	·011
6	·004

Now, by inspection of the curve it is clear that in any current ring of one centimetre width, that part of it nearer to the point O produces a much stronger field at O than its more remote portion. For instance, in the belt or ring of current between the points 1 and 2 the perpendicular line proportional to the field is much higher at 1 than 2, and if we imagine this ring subdivided into a very large number of small rings, it is easy to see that the field of each (at the point O) gradually becomes more intense as they are further from 2 and nearer to 1, the increase being in exact proportion to the rise of the curve. In fact, every perpendicular line drawn from the axis between 1 and 2 to meet the curve is

proportional to the field produced at O by a narrow ring of current *in the plane of that line;* and in order to estimate the total field produced by the belt of current between 1 and 2, it is clear that we must add together the values of as many perpendicular lines as can be crowded into that space. But this is the same thing as finding the *area* of the strip included between the two vertical lines at 1 and 2, the curve, and the axis. Now the area of this strip we may consider as made up of two parts—a rectangle and a space like a triangle (shown separated by a dotted line). It would be very easy to find the area of the rectangle, but not so easy to find the area of the triangular space, because one side is curved. But by dividing the space between 1 and 2 into a series of smaller strips the curved side of the triangle in each might be considered straight and the areas found by inspection, adding them together afterwards to get the area of the whole strip. Fortunately we are not obliged to do anything so laborious as that, for by considering any very narrow strip, such as that drawn on the off-side of the figure 2, it will be seen that the little triangle becomes very diminutive, and finally disappears altogether if we take the strip smaller and smaller *without limit.* Just in the same way as the field produced by the belt of current between 1 and 2 is found by measuring the area of the strip contained between these limits, so the total field at the point O due to the whole solenoid is found by calculating the area of the whole curve between the limits O and l. By taking such extremely small strips, then, we may regard them as rectangles of minute breadth, and adding them all together from O to l gives the total area of the curve. It is not necessary to understand the exact process by which this is done so long as the student understands *what* is done and *why* it is done. It will be understood that in adding any series of numbers the process of addition is much simplified if we first separate any factors which are *constant* from those which *vary.* For instance, suppose it is required to sum up all multiples of some number which we will call a, between the limits of $5a$ and $9a$, it is easier to perform the summation by adding the factors which

vary first; thus:—$5+6+7+8+9=35$, and then taking the product $35a$, than to add together $5a+6a+7a+8a+9a$. The difference in the labour involved in doing it these two ways is at once apparent by taking a as some fractional number, say $1\frac{1}{7}$. In other words, $a=1\frac{1}{7}$ is *constant* throughout, and it is only necessary to sum the factors which *vary* as above, and then multiply their sum by the constant, giving the final summation of the series, viz., $1\frac{1}{7}\times 35=40$. Similarly, in the summation of the little rectangles representing the fields produced by each narrow ring of current there are two factors which go to make up the value of each rectangle, one being constant and the other varying with the position of the current ring. The constant factor in this series is the product $2\,\pi\,C_1\,r^2$, and the factor which varies comprises the reciprocal of R^3 (which

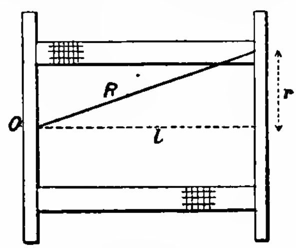


FIG. 149.

varies with the position of each ring), and a fractional quantity which diminishes the width of the current rings (and therefore of the rectangles) down to an infinitesimally small amount. This latter fraction would be a constant quantity were it not for the fact that it is expressed as a fraction of the distance of each ring from the point where the field is to be determined, and this distance of course is different for every ring. We have now only to take the factors which vary and sum up all their possible values between the limits O and l (the length of the solenoid), the result of this summation being equal to

$$\frac{l}{r^2\,R},$$

where R has its greatest value, viz., the distance from the point O to the mean depth of the furthest ring (*see* Fig. 149).

The depth of the winding is represented in the figure by the space (partially shaded) between the two bounding lines of the coil, and the line R is from O to a point at the side of the coil central to the winding—in fact, to the point which limits the mean radius r. Multiplying the constant quantity by the sum of the factors which vary, we have the final summation, giving the total field at O due to the entire solenoid, viz. :—

$$2\pi C_1 r^2 \times \frac{l}{r^2 R} \text{ C.-G.-S. units,}$$

which, after cancelling out r^2, becomes

$$2\pi C_1 \times \frac{l}{R} \text{ C.-G.-S. units.}$$

Putting for C_1 its equivalent (Cn divided by l), and cancelling out l, we have for the field at either end of the solenoid

$$\frac{2\pi Cn}{R} \text{ C.-G.-S. units.}$$

Let us now apply this to the numerical example given above. The length of R will be seen to be the square root of the sum of the squares of the mean radius and length of the solenoid, which are two and six centimetres respectively. Hence

$$R = \sqrt{4+36} = 6{\cdot}32 \text{ centimetres.}$$

The current is 3 amperes (or ·3 C.-G.-S. units), and the number of turns of wire 200; therefore the field at either end of this solenoid is equal to

$$\frac{2 \times 3{\cdot}1416 \times {\cdot}3 \times 200}{6{\cdot}32} = \frac{377}{6{\cdot}32} = 59{\cdot}6 \text{ C.-G.-S. units.}$$

Now suppose we take a solenoid (Fig 150) exactly double the length of the above, but in all other respects the same, that is having the same depth of winding and the same current. Evidently the field at the point O, taken now at the centre, will be twice that at the end of the former solenoid, and therefore we can write it down at once as

$$4\pi C_1 \times \frac{l}{R} \text{ C.-G.-S. units.}$$

But l is now only half the length of this solenoid, and therefore it is advisable to multiply both l and R by 2 (which does not alter their ratio). Now the value 2 R is equal to the line connecting opposite corners of the solenoid (Fig. 150), which we will distinguish by calling it the diagonal (D). And instead of writing $2\,l$ for the length of the present soleuoid we shall write it l, keeping this letter to represent the length of solenoid whatever it may be. Therefore the field at the centre becomes

$$4\,\pi\, C_1 \times \frac{l}{D} \text{ C.-G.-S. units.}$$

Now let C_1 (the current per unit length) be expressed in terms of C (the current passing through the wire). It will be understood that the current per unit length of coil is the same

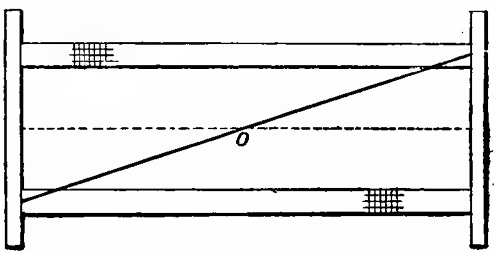


Fig. 150.

value as before, and is expressed by the total current flowing round the bobbin ($C\,n$) divided by l, n being the number of turns and l the length of the present solenoid. Cancelling out l the field at O becomes

$$\frac{4\,\pi\, C\, n}{D} \text{ C.-G.-S. units.}$$

This is applicable to any dimensions of solenoid, and gives the field produced at the central point on the axis where it is most frequently required to be known.

We have now the means of determining the strength of field at either end of a solenoid and at its centre on the axis. It will be understood that, taking *any* size solenoid whatever, and

denoting by the letters R and D the two lines drawn and so marked in the figure (Fig. 151), that the field at either end on the axis is equal to

$$\frac{2\,\pi\,C\,n}{R}\ \text{C.-G.-S. units.}$$

and that at the centre is

$$\frac{4\,\pi\,C\,n}{D}\ \text{C.-G.-S. units.}$$

It is worthy of note that when the solenoid is very long R becomes practically equal to D, while if the solenoid is very short, more like a galvanometer coil, R is practically equal to half of D. Applying the latter expression to find the field at the centre of

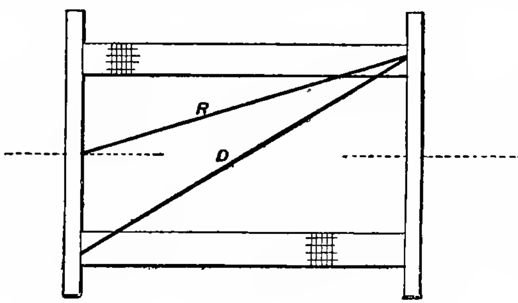


Fig. 151.

the solenoid whose dimensions are given above (Fig. 148), we find that the numerator is twice the previously calculated amount, and is therefore equal to 754. The length of D is the square root of the sum of the squares of the mean diameter and length of solenoid, which are 4 and 6 centimetres respectively. Hence

$$D = \sqrt{16+36} = 7{\cdot}21 \text{ centimetres,}$$

and the field at the centre is therefore equal to

$$\frac{754}{7{\cdot}21} = 104{\cdot}5 \text{ C.-G.-S. units.}$$

being somewhat less than double that at each end,

It will easily be seen that the *longer* the solenoid the nearer does D become equal to l, and that for a long narrow solenoid we may consider it the same as l without any appreciable error. The field at the centre of such a solenoid is then—

$$\frac{4\pi C n}{l} \text{ C.-G.-S. units,}$$

to which expression reference has already been made (paras. 130 and 133). Again, the shorter the length of the solenoid the more it approaches the shape of a current ring (Fig. 152), and we should therefore expect the field at the centre to become very nearly equal to that due to a thin ring of current. This is so, for the diagonal line is very nearly the same as the mean diameter of the coil, and does become exactly equal to it when the coil is made shorter and shorter till it forms a narrow ring. When such is the case, instead of the diagonal D in the above expression we can put the mean diameter of the coil or twice the mean radius ($2\,r$). Cancelling out the 2, the field at the centre becomes

$$\frac{2\,\pi\,C\,n}{r} \text{ C.-G.-S. units,}$$

which is the same as previously derived for a narrow ring-shaped coil of n turns (para. 145).

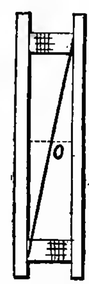


FIG. 152.

The next point to consider is the strength of field at *any* point on the axis either inside or outside the solenoid, and this case will necessarily include those already taken. The expressions, however, which have been derived above for finding the field at the centre and at either end of a solenoid will be found useful not only as steps leading up to the most general case, but as easy expressions to deal with, showing also at a glance the relation between the field at these points and the diameter, length, and depth of winding of a solenoid.

150. Intensity of Field at any Point on the Axis of a Solenoid.—The results already derived will afford sufficient ground to work upon in order to show how the field at any point on

the axis of a solenoid is determined. In the two expressions for the field at the centre and at either end, if we consider only those factors which alter in value according to the position of the point on the axis, we shall be able to trace the general law for any point. On examining these expressions we find that the factor $2\pi C_1$ occurs in both, and therefore the remaining factors depending on the position of the point considered are

$$\frac{2l}{D} \text{ at the centre,}$$

and

$$\frac{l}{R} \text{ at either end.}$$

Now the ratio of the length of the solenoid (l) to the diagonal

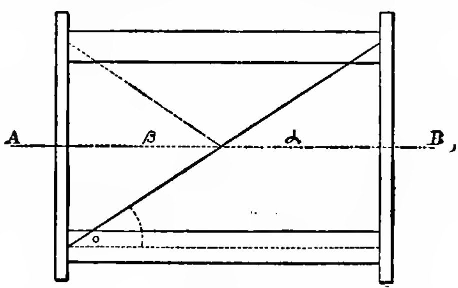


FIG. 153.

(D) is the cosine of the angle (0) between them (Fig. 153), and it is evident that the angle 0 is equivalent to the angle α. Further, if a line is drawn from the centre to the other end of the solenoid, as shown, it is evident that the included angle β is symmetrical with the angle α and equal to it.

We have, therefore,

$$\frac{2l}{D} = 2\cos\alpha = \cos\alpha + \cos\beta.$$

This is for the field at the centre of the coil; and now if we consider the field at one end we find in a similar manner that the ratio of l to R (Fig. 149) is the cosine of the angle between these lines. Generalising these results, it is not difficult

to see that the field along the axis *within* the coil depends upon the sum of the cosines of the angles formed on each side of the point wherever that is situated on the axis. For instance, as the field is considered at points further away from the centre one angle becomes less and less while the other becomes greater; and finally, when the point is taken at the end of the coil, as in Fig. 149, the angle β has increased to 90deg. where its cosine is zero, while α has become equal to the angle enclosed between the lines l and R, as we have seen above.

Replacing the constant factor $2\pi C_1$, and putting for C_1 its equivalent ($C n$ divided by l), we have the result that the

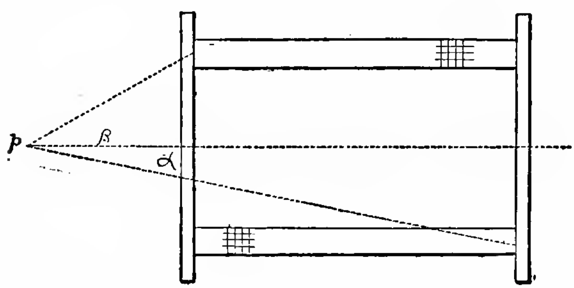


FIG. 154.

intensity of field at any point on the axis *within* the coil is equal to

$$\frac{2\pi C n}{l}(\cos\alpha + \cos\beta) \text{ C.-G.-S. units,}$$

where at the end of the coil the cosine of β is zero, and beyond this, viz., *outside* the coil, its cosine changes in sign. The field at any point outside the coil on its axis (such as at p, Fig. 154) is, therefore, proportional to the *difference* of the cosines of the two angles instead of their sum as above. (The angle α in this figure is shown on the other side of the axis for convenience. It is evident that it is identically the same as when indicated above the axis, in the same way, at the point p.) This may now be applied to some practical example. It is best to take some definite

size of solenoid and calculate the intensity of field at various points along the axis, and from these results plot a curve showing the way in which the field varies. By this means an insight is obtained into the relative strength of field at various points on the axis of such a coil. Take a solenoid eight centimetres long and wound with 100 turns of wire, the mean radius of the coil so wound being one centimetre (Fig. 155). Say a current of 12·7

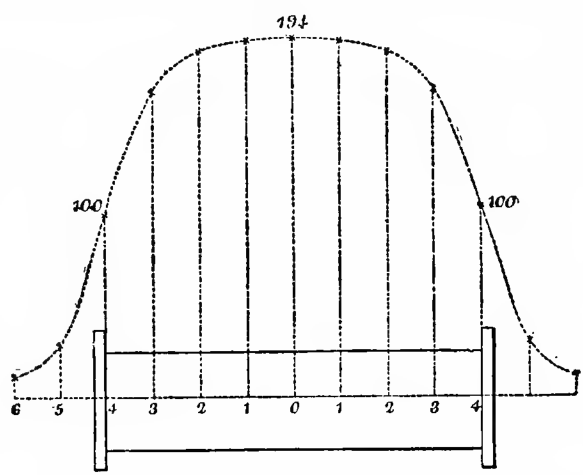


Fig. 155.

amperes is passed through this winding, then the constant quantity

$$\frac{2\,\pi\,C\,n}{l} = \frac{2 \times 3{\cdot}1416 \times 1{\cdot}27 \times 100}{8} = 100,$$

the current being expressed in C.-G.-S. units. The factors which vary according to the position of the points on the axis have now to be determined, and as the distribution of field is symmetrical on each side of the centre it is only necessary to make the calculations for one side. The angles themselves need not be determined, but simply their cosines, and this only involves in each case finding the value of the hypothenuse of a

right angle triangle of which the other two sides are known. We have at the centre two equal angles, the cosine of each of which is

$$\frac{4}{\sqrt{4^2+1}} = \cdot 97,$$

the denominator being the hypothenuse, and found by extracting the root of the sum of the squares of the other two sides. At the centre of the coil the field is, therefore,

$$100 \times 2 \times \cdot 97 = 194 \text{ C.-G.-S. units.}$$

Now calculate the field at points on the axis one centimetre apart. At one centimetre distance from the centre the two angles are different, their cosines being found in the same way as the above, and added together, viz. :—

$$\frac{5}{\sqrt{26}} + \frac{3}{\sqrt{10}} = 1 \cdot 93.$$

The field at this point is, therefore,

$$100 \times 1 \cdot 93 = 193 \text{ C.-G.-S. units.}$$

Similarly, at two centimetres from the centre the field is

$$100\left(\frac{6}{\sqrt{37}} + \frac{2}{\sqrt{5}}\right) = 188 \text{ C.-G.-S. units.}$$

At three centimetres the field, similarly calculated, is equal to 169 units. Now at four centimetres from the centre we arrive at the end of the coil where one angle becomes 90 deg. and its cosine nil, the cosine of the remaining angle being

$$\frac{8}{\sqrt{65}} = \cdot 99.$$

The field at the end of the coil is, therefore,

$$100 \times \cdot 99 = 99 \text{ C.-G.-S units,}$$

or a little more than half that at the centre. It will be observed that the field diminishes very rapidly at this part of

the axis. At five centimetres from the centre the point is outside the coil, and the *difference* of the cosines is therefore taken. The field is therefore

$$100\left(\frac{9}{\sqrt{82}}-\frac{1}{\sqrt{2}}\right)=28 \text{ C.-G.-S. units.}$$

Similarly, at six centimetres from the centre the field is equal to

$$100\left(\frac{10}{\sqrt{101}}-\frac{2}{\sqrt{5}}\right)=10 \text{ C.-G.-S. units.}$$

These values plotted give the curve shown in the figure, which therefore graphically represents the variation in intensity of field along the axis. It will also be noticed by the foregoing example that the variation of field along the axis depends only upon the geometrical dimensions of the coil; that is, on the relation between the mean diameter and length of the solenoid, and not on its actual size as a whole. For example, the curve above plotted (Fig. 155) is for a solenoid whose length is four times its mean diameter, and there would be precisely the same distribution of field and consequently the same shape of curve for solenoids of 2, 3, 10, or any number of times the *actual* size of the above, provided they have the same relation between length and diameter; and this is irrespective of the strength of current, number of turns, and size of wire composing the coil. The above does not profess to be a proof of the law stated, but the endeavour has been to trace up in a rational manner the derivation of the formula, so that it may be used with an intelligent appreciation of its meaning.

TABLE OF NATURAL TANGENTS.

The tangent of 45° being exactly equal to unity, the tangents of angles under 45 are fractional quantities, while the tangents of angles above 45 are greater than unity, but not in exact whole numbers.

The fractional quantities are given in the vertical columns, each such fraction corresponding to the degree marked in the same horizontal line in the first column and the number of "minutes" marked at the head of the column in which the fractional quantity is placed.

Instead of 60 vertical columns, viz., one for each minute, it is only for every sixth minute that the fraction is given in full. The last two columns supply figures for the intermediate five minutes by which the fractional numbers must be increased.

If the angle is simply in degrees its complete tangent is at once found in the second column. In all other columns the whole numbers are omitted, but are understood to be the same as those marked in the second column in the same horizontal line, except when the first figure of the fraction has a mark over it, when the next higher whole number is to be taken.

With the aid of a few examples no difficulty will be experienced in using the table.

Required the tangent of 36° 45′ :—

By the table—tangent of 36° 42′ = ·7454
Difference in Minutes = 3
Number under 3.................. = 14

Add................................·7468 = required tan.

Required the tangent of 71° 50′ :—

By the table—tangent of 71° 48′ = 3·0415
Difference in Minutes = 2
Number under 2.................. 58

Add................................3·0473 = required tan.

Required the tangent of 83° 54′ :—

By the table—tangent of 83° 54′ = 9·3572

Example of the reverse operation :

Required the angle whose tangent is 2·3090 :—

Nearest lower fractional number opposite whole number 2	= ·2998 = tan	66° 30′
Difference between this and 3090 =	92	
In difference column 92 corresponds to		5′
Add................................		66° 35′

which is the required angle.

	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1	2	3	4	5
0°	·0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	14
1	·0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	·0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	3	6	9	12	15
3	0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	3	6	9	12	15
4	·0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	3	6	9	12	15
5	·0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	3	6	9	12	15
6	·1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	3	6	9	12	15
7	·1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	3	6	9	12	15
8	·1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	3	6	9	12	15
9	·1584	1602	1620	1638	1655	1673	1691	1709	1727	1745	3	6	9	12	15
10	·1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	3	6	9	12	15
11	·1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	3	6	9	12	15
12	·2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	3	6	9	12	15
13	·2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	3	6	9	12	15
14	·2493	2512	2530	2549	2568	2586	2605	2623	2642	2661	3	6	9	12	16
15	·2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	3	6	9	13	16
16	·2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3	6	9	13	16
17	·3057	3076	3096	3115	3134	3153	3172	3191	3211	3230	3	6	10	13	15
	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1	2	3	4	5

	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1	2	3	4	5
18°	·3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3	6	10	13	16
19	·3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	3	6	10	13	17
20	·3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	3	7	10	13	17
21	·3839	3859	3879	3899	3919	3939	3959	3979	4000	4020	3	7	10	13	17
22	·4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	3	7	10	14	17
23	·4245	4265	4286	4307	4327	4348	4369	4390	4411	4431	3	7	10	14	17
24	·4452	4473	4494	4515	4536	4557	4578	4599	4621	4642	4	7	10	14	18
25	·4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4	7	11	14	18
26	·4877	4899	4921	4942	4964	4986	5008	5029	5051	5073	4	7	11	15	18
27	·5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	4	7	11	15	18
28	·5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	4	8	11	15	19
29	·5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	4	8	12	15	19
30	·5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	4	8	12	16	20
31	·6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	4	8	12	16	20
32	·6249	6273	6297	6322	6346	6371	6395	6420	6445	6469	4	8	12	16	20
33	·6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	4	8	13	17	21
34	·6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	4	9	13	17	21
35	·7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	4	9	13	18	22
	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1	2	3	4	5

	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1	2	3	4	5
36°	·7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	5	9	14	18	23
37	·7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	5	9	14	18	23
38	·7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	5	10	14	19	24
39	·8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	5	10	15	20	24
40	·8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	5	10	15	20	25
41	·8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	5	10	16	21	26
42	·9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	5	11	16	21	27
43	·9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	6	11	17	22	28
44	·9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	6	11	17	23	29
45	1·0000	0035	0070	0105	0141	0176	0212	0247	0283	0319	6	12	18	24	30
46	1·0355	0392	0428	0464	0501	0538	0575	0612	0649	0686	6	12	18	25	31
47	1·0724	0761	0799	0837	0875	0913	0951	0990	1028	1067	6	13	19	25	32
48	1·1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	7	13	20	26	33
49	1·1504	1544	1585	1626	1667	1708	1750	1792	1833	1875	7	14	21	28	34
50	1·1918	1960	2002	2045	2088	2131	2174	2218	2261	2305	7	14	22	29	36
51	1·2349	2393	2437	2482	2527	2572	2617	2662	2708	2753	8	15	23	30	38
52	1·2799	2846	2892	2938	2985	3032	3079	3127	3175	3222	8	16	23	31	39
53	1·3270	3319	3367	3416	3465	3514	3564	3613	3663	3713	8	16	25	33	41
	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1	2	3	4	5

	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1	2	3	4	5
54°	1·3764	3814	3865	3916	3968	4019	4071	4124	4176	4229	9	17	26	34	43
55	1·4281	4335	4388	4442	4496	4550	4605	4659	4715	4770	9	18	27	36	45
56	1·4826	4882	4938	4994	5051	5108	5166	5224	5282	5340	10	19	29	38	48
57	1·5399	5458	5517	5577	5637	5697	5757	5818	5880	5941	10	20	30	40	50
58	1·6003	6066	6128	6191	6255	6319	6383	6447	6512	6577	11	21	32	43	53
59	1·6643	6709	6775	6842	6909	6977	7045	7113	7182	7251	11	23	34	45	56
60	1·7321	7391	7461	7532	7603	7675	7747	7820	7893	7966	12	24	36	48	60
61	1·8040	8115	8190	8265	8341	8418	8495	8572	8650	8728	13	26	38	51	64
62	1·8807	8887	8967	9047	9128	9210	9292	9375	9458	9542	14	27	41	55	68
63	1·9626	9711	9797	9883	9970	0057	$\bar{0}145$	$\bar{0}233$	$\bar{0}323$	$\bar{0}413$	15	29	44	58	73
64	2·0503	0594	0686	0778	0872	0965	1060	1155	1251	1348	16	31	47	63	78
65	2·1445	1543	1642	1742	1842	1943	2045	2148	2251	2355	17	34	51	68	85
66	2·2460	2566	2673	2781	2889	2998	3109	3220	3332	3445	18	37	55	74	92
67	2·3559	3673	3789	3906	4023	4142	4262	4383	4504	4627	20	40	60	79	99
68	2·4751	4876	5002	5129	5257	5386	5517	5649	5782	5916	22	43	65	87	108
69	2·6051	6187	6325	6464	6605	6746	6889	7034	7179	7326	24	47	71	95	118
70	2·7475	7625	7776	7929	8083	8239	8397	8556	8716	8878	26	52	78	104	130
71	2·9042	9208	9375	9544	9714	9887	0061	0237	$\bar{0}415$	$\bar{0}595$	29	58	87	115	144
	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1	2	3	4	5

	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1	2	3	4	5
72°	3·0777	0961	1146	1334	1524	1716	1910	2106	2305	2506	32	64	96	129	161
73	3·2709	2914	3122	3332	3544	3759	3977	4197	4420	4646	36	72	108	144	180
74	3·4874	5105	5339	5576	5816	6059	6305	6554	6806	7062	41	81	122	162	203
75	3·7321	7583	7848	8118	8391	8667	8947	9232	9520	9812	46	93	139	186	232
76	4·0108	0408	0713	1022	1335	1653	1976	2303	2635	2972	53	107	160	213	267
77	4·3315	3662	4015	4374	4737	5107	5483	5864	6252	6646	62	124	186	248	310
78	4·7046	7453	7867	8288	8716	9152	9594	$\bar{0}045$	0504	$\bar{0}970$	73	146	219	292	365
79	5·1446	1929	2422	2924	3435	3955	4486	5026	5577	6140	87	175	262	350	437
80	5·6713	7297	7894	8502	9124	9758	0405	1066	$\bar{1}742$	$\bar{2}432$	Difference columns cease to be useful, owing to the rapidity with which the value of the tangent changes.				
81	6·3138	3859	4596	5350	6122	6912	7920	8548	9395	$\bar{0}264$					
82	7·1154	2066	3002	3962	4947	5958	6996	$\bar{8}062$	$\bar{9}153$	$\bar{0}285$					
83	8·1443	2636	3863	5126	6427	7769	9152	0579	$\bar{2}052$	$\bar{3}572$					
84	9·5144	9·677	9·845	10·02	10·20	10·39	10·58	10·78	10·99	11·20					
85	11·43	11·66	11·91	12·16	12·43	12·71	13·00	13·30	13·62	13·95					
86	14·30	14·67	15·06	15·46	15·89	16·35	16·83	17·34	17·89	18·46					
87	19·08	19·74	20·45	21·20	22·02	22·90	23·86	24·90	26·03	27·27					
88	28·64	30·14	31·82	33·69	35·80	38·19	40·92	44·07	47·74	52·08					
89	57·29	63·66	71·62	81·85	95·49	114·6	143·2	191·0	286·5	573·0					
	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1	2	3	4	5

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